

Maple 2018.2 Integration Test Results
on the problems in "6 Hyperbolic functions/6.2 Hyperbolic cosine"

Test results for the 48 problems in "6.2.1 (c+d x)^m (a+b cosh)^n.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^4 \cosh(bx+a) dx$$

Optimal(type 3, 91 leaves, 5 steps):

$$-\frac{24d^3(dx+c)\cosh(bx+a)}{b^4} - \frac{4d(dx+c)^3\cosh(bx+a)}{b^2} + \frac{24d^4\sinh(bx+a)}{b^5} + \frac{12d^2(dx+c)^2\sinh(bx+a)}{b^3} + \frac{(dx+c)^4\sinh(bx+a)}{b}$$

Result(type 3, 546 leaves):

$$\begin{aligned} & \frac{1}{b} \left(c^4 \sinh(bx+a) - \frac{12d^3ac((bx+a)^2\sinh(bx+a) - 2(bx+a)\cosh(bx+a) + 2\sinh(bx+a))}{b^3} \right. \\ & + \frac{12d^3a^2c((bx+a)\sinh(bx+a) - \cosh(bx+a))}{b^3} - \frac{12d^2a^2c^2((bx+a)\sinh(bx+a) - \cosh(bx+a))}{b^2} \\ & + \frac{d^4((bx+a)^4\sinh(bx+a) - 4(bx+a)^3\cosh(bx+a) + 12(bx+a)^2\sinh(bx+a) - 24(bx+a)\cosh(bx+a) + 24\sinh(bx+a))}{b^4} \\ & + \frac{d^4a^4\sinh(bx+a)}{b^4} - \frac{4d^3a^3c\sinh(bx+a)}{b^3} + \frac{6d^2a^2c^2\sinh(bx+a)}{b^2} - \frac{4dac^3\sinh(bx+a)}{b} \\ & - \frac{4d^4a((bx+a)^3\sinh(bx+a) - 3(bx+a)^2\cosh(bx+a) + 6(bx+a)\sinh(bx+a) - 6\cosh(bx+a))}{b^4} \\ & + \frac{4d^3c((bx+a)^3\sinh(bx+a) - 3(bx+a)^2\cosh(bx+a) + 6(bx+a)\sinh(bx+a) - 6\cosh(bx+a))}{b^3} \\ & + \frac{6d^4a^2((bx+a)^2\sinh(bx+a) - 2(bx+a)\cosh(bx+a) + 2\sinh(bx+a))}{b^4} \\ & + \frac{6d^2c^2((bx+a)^2\sinh(bx+a) - 2(bx+a)\cosh(bx+a) + 2\sinh(bx+a))}{b^2} - \frac{4d^4a^3((bx+a)\sinh(bx+a) - \cosh(bx+a))}{b^4} \\ & \left. + \frac{4dc^3((bx+a)\sinh(bx+a) - \cosh(bx+a))}{b} \right) \end{aligned}$$

Problem 2: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^2 \cosh(bx+a) dx$$

Optimal(type 3, 49 leaves, 3 steps):

$$-\frac{2d(dx+c)\cosh(bx+a)}{b^2} + \frac{2d^2\sinh(bx+a)}{b^3} + \frac{(dx+c)^2\sinh(bx+a)}{b}$$

Result(type 3, 146 leaves):

$$\frac{1}{b} \left(\frac{d^2 \left((bx+a)^2 \sinh(bx+a) - 2(bx+a) \cosh(bx+a) + 2 \sinh(bx+a) \right)}{b^2} - \frac{2d^2 a \left((bx+a) \sinh(bx+a) - \cosh(bx+a) \right)}{b^2} \right. \\ \left. + \frac{2dc \left((bx+a) \sinh(bx+a) - \cosh(bx+a) \right)}{b} + \frac{d^2 a^2 \sinh(bx+a)}{b^2} - \frac{2dac \sinh(bx+a)}{b} + c^2 \sinh(bx+a) \right)$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(bx+a)}{(dx+c)^3} dx$$

Optimal(type 4, 96 leaves, 5 steps):

$$\frac{b^2 \operatorname{Chi}\left(\frac{bc}{d} + bx\right) \cosh\left(a - \frac{bc}{d}\right)}{2d^3} - \frac{\cosh(bx+a)}{2d(dx+c)^2} + \frac{b^2 \operatorname{Shi}\left(\frac{bc}{d} + bx\right) \sinh\left(a - \frac{bc}{d}\right)}{2d^3} - \frac{b \sinh(bx+a)}{2d^2(dx+c)}$$

Result(type 4, 276 leaves):

$$\frac{b^3 e^{-bx-a} x}{4d(b^2 d^2 x^2 + 2b^2 c dx + c^2 b^2)} + \frac{b^3 e^{-bx-a} c}{4d^2(b^2 d^2 x^2 + 2b^2 c dx + c^2 b^2)} - \frac{b^2 e^{-bx-a}}{4d(b^2 d^2 x^2 + 2b^2 c dx + c^2 b^2)} - \frac{b^2 e^{-\frac{da-cb}{d}} \operatorname{Ei}_1\left(bx+a - \frac{da-cb}{d}\right)}{4d^3} \\ - \frac{b^2 e^{bx+a}}{4d^3 \left(\frac{bc}{d} + bx\right)^2} - \frac{b^2 e^{bx+a}}{4d^3 \left(\frac{bc}{d} + bx\right)} - \frac{b^2 e^{\frac{da-cb}{d}} \operatorname{Ei}_1\left(-bx-a - \frac{-da+cb}{d}\right)}{4d^3}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^2 \cosh(bx+a)^2 dx$$

Optimal(type 3, 85 leaves, 4 steps):

$$\frac{d^2 x}{4b^2} + \frac{(dx+c)^3}{6d} - \frac{d(dx+c) \cosh(bx+a)^2}{2b^2} + \frac{d^2 \cosh(bx+a) \sinh(bx+a)}{4b^3} + \frac{(dx+c)^2 \cosh(bx+a) \sinh(bx+a)}{2b}$$

Result(type 3, 261 leaves):

$$\frac{1}{b} \left(\frac{d^2 \left(\frac{(bx+a)^2 \cosh(bx+a) \sinh(bx+a)}{2} + \frac{(bx+a)^3}{6} - \frac{(bx+a) \cosh(bx+a)^2}{2} + \frac{\cosh(bx+a) \sinh(bx+a)}{4} + \frac{bx}{4} + \frac{a}{4} \right)}{b^2} \right. \\ \left. - \frac{2d^2 a \left(\frac{(bx+a) \cosh(bx+a) \sinh(bx+a)}{2} + \frac{(bx+a)^2}{4} - \frac{\cosh(bx+a)^2}{4} \right)}{b^2} \right)$$

$$\begin{aligned}
& + \frac{2dc \left(\frac{(bx+a) \cosh(bx+a) \sinh(bx+a)}{2} + \frac{(bx+a)^2}{4} - \frac{\cosh(bx+a)^2}{4} \right)}{b} + \frac{d^2 a^2 \left(\frac{\cosh(bx+a) \sinh(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right)}{b^2} \\
& - \frac{2dac \left(\frac{\cosh(bx+a) \sinh(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right)}{b} + c^2 \left(\frac{\cosh(bx+a) \sinh(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2} \right)
\end{aligned}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(bx+a)^2}{(dx+c)^4} dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$\begin{aligned}
& \frac{b^2}{3d^3(dx+c)} - \frac{\cosh(bx+a)^2}{3d(dx+c)^3} - \frac{2b^2 \cosh(bx+a)^2}{3d^3(dx+c)} + \frac{2b^3 \cosh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{2b^3 \text{Chi}\left(\frac{2bc}{d} + 2bx\right) \sinh\left(2a - \frac{2bc}{d}\right)}{3d^4} \\
& - \frac{b \cosh(bx+a) \sinh(bx+a)}{3d^2(dx+c)^2}
\end{aligned}$$

Result (type 4, 554 leaves):

$$\begin{aligned}
& - \frac{1}{6d(dx+c)^3} - \frac{b^5 e^{-2bx-2a} x^2}{6d(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + b^3 c^3)} - \frac{b^5 e^{-2bx-2a} cx}{3d^2(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + b^3 c^3)} \\
& - \frac{b^5 e^{-2bx-2a} c^2}{6d^3(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + b^3 c^3)} + \frac{b^4 e^{-2bx-2a} x}{12d(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + b^3 c^3)} \\
& + \frac{b^4 e^{-2bx-2a} c}{12d^2(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + b^3 c^3)} - \frac{b^3 e^{-2bx-2a}}{12d(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + b^3 c^3)} \\
& + \frac{b^3 e^{-\frac{2(da-cb)}{d}} \text{Ei}_1\left(2bx+2a - \frac{2(da-cb)}{d}\right)}{3d^4} - \frac{b^3 e^{2bx+2a}}{12d^4\left(\frac{bc}{d} + bx\right)^3} - \frac{b^3 e^{2bx+2a}}{12d^4\left(\frac{bc}{d} + bx\right)^2} - \frac{b^3 e^{2bx+2a}}{6d^4\left(\frac{bc}{d} + bx\right)} \\
& - \frac{b^3 e^{\frac{2(da-cb)}{d}} \text{Ei}_1\left(-2bx-2a - \frac{2(-da+cb)}{d}\right)}{3d^4}
\end{aligned}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int (dx+c)^4 \cosh(bx+a)^3 dx$$

Optimal (type 3, 205 leaves, 12 steps):

$$\begin{aligned}
& - \frac{160 d^3 (dx+c) \cosh(bx+a)}{9 b^4} - \frac{8 d (dx+c)^3 \cosh(bx+a)}{3 b^2} - \frac{8 d^3 (dx+c) \cosh(bx+a)^3}{27 b^4} - \frac{4 d (dx+c)^3 \cosh(bx+a)^3}{9 b^2} + \frac{488 d^4 \sinh(bx+a)}{27 b^5} \\
& + \frac{80 d^2 (dx+c)^2 \sinh(bx+a)}{9 b^3} + \frac{2 (dx+c)^4 \sinh(bx+a)}{3 b} + \frac{4 d^2 (dx+c)^2 \cosh(bx+a)^2 \sinh(bx+a)}{9 b^3} \\
& + \frac{(dx+c)^4 \cosh(bx+a)^2 \sinh(bx+a)}{3 b} + \frac{8 d^4 \sinh(bx+a)^3}{81 b^5}
\end{aligned}$$

Result(type 3, 1216 leaves):

$$\begin{aligned}
& \frac{1}{b} \left(\frac{1}{b^4} \left(d^4 \left(\frac{2 (bx+a)^4 \sinh(bx+a)}{3} + \frac{(bx+a)^4 \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{28 (bx+a)^3 \cosh(bx+a)}{9} + \frac{80 (bx+a)^2 \sinh(bx+a)}{9} \right. \right. \right. \\
& - \frac{488 (bx+a) \cosh(bx+a)}{27} + \frac{1456 \sinh(bx+a)}{81} - \frac{4 (bx+a)^3 \sinh(bx+a)^2 \cosh(bx+a)}{9} + \frac{4 (bx+a)^2 \sinh(bx+a) \cosh(bx+a)^2}{9} \\
& - \left. \left. \frac{8 (bx+a) \sinh(bx+a)^2 \cosh(bx+a)}{27} + \frac{8 \cosh(bx+a)^2 \sinh(bx+a)}{81} \right) \right) - \frac{1}{b^4} \left(4 d^4 a \left(\frac{2 (bx+a)^3 \sinh(bx+a)}{3} \right. \right. \\
& + \frac{(bx+a)^3 \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{7 (bx+a)^2 \cosh(bx+a)}{3} + \frac{40 (bx+a) \sinh(bx+a)}{9} - \frac{122 \cosh(bx+a)}{27} \\
& - \left. \left. \frac{(bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a)}{3} + \frac{2 (bx+a) \sinh(bx+a) \cosh(bx+a)^2}{9} - \frac{2 \sinh(bx+a)^2 \cosh(bx+a)}{27} \right) \right) \\
& + \frac{1}{b^4} \left(6 d^4 a^2 \left(\frac{(bx+a)^2 \sinh(bx+a) \cosh(bx+a)^2}{3} + \frac{2 (bx+a)^2 \sinh(bx+a)}{3} - \frac{2 (bx+a) \sinh(bx+a)^2 \cosh(bx+a)}{9} \right. \right. \\
& - \left. \left. \frac{14 (bx+a) \cosh(bx+a)}{9} + \frac{2 \cosh(bx+a)^2 \sinh(bx+a)}{27} + \frac{40 \sinh(bx+a)}{27} \right) \right) \\
& - \frac{4 d^4 a^3 \left(\frac{2 (bx+a) \sinh(bx+a)}{3} + \frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{7 \cosh(bx+a)}{9} - \frac{\sinh(bx+a)^2 \cosh(bx+a)}{9} \right)}{b^4} \\
& + \frac{d^4 a^4 \left(\frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \sinh(bx+a)}{b^4} + \frac{1}{b^3} \left(4 c d^3 \left(\frac{2 (bx+a)^3 \sinh(bx+a)}{3} + \frac{(bx+a)^3 \sinh(bx+a) \cosh(bx+a)^2}{3} \right. \right. \\
& - \left. \left. \frac{7 (bx+a)^2 \cosh(bx+a)}{3} + \frac{40 (bx+a) \sinh(bx+a)}{9} - \frac{122 \cosh(bx+a)}{27} - \frac{(bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a)}{3} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{2 (b x + a) \sinh(b x + a) \cosh(b x + a)^2}{9} - \frac{2 \sinh(b x + a)^2 \cosh(b x + a)}{27} \Big) - \frac{1}{b^3} \left(12 c d^3 a \left(\frac{(b x + a)^2 \sinh(b x + a) \cosh(b x + a)^2}{3} \right. \right. \\
& + \frac{2 (b x + a)^2 \sinh(b x + a)}{3} - \frac{2 (b x + a) \sinh(b x + a)^2 \cosh(b x + a)}{9} - \frac{14 (b x + a) \cosh(b x + a)}{9} + \frac{2 \cosh(b x + a)^2 \sinh(b x + a)}{27} \\
& \left. \left. + \frac{40 \sinh(b x + a)}{27} \right) \right) \\
& + \frac{12 c d^3 a^2 \left(\frac{2 (b x + a) \sinh(b x + a)}{3} + \frac{(b x + a) \sinh(b x + a) \cosh(b x + a)^2}{3} - \frac{7 \cosh(b x + a)}{9} - \frac{\sinh(b x + a)^2 \cosh(b x + a)}{9} \right)}{b^3} \\
& - \frac{4 c d^3 a^3 \left(\frac{2}{3} + \frac{\cosh(b x + a)^2}{3} \right) \sinh(b x + a)}{b^3} + \frac{1}{b^2} \left(6 c^2 d^2 \left(\frac{(b x + a)^2 \sinh(b x + a) \cosh(b x + a)^2}{3} + \frac{2 (b x + a)^2 \sinh(b x + a)}{3} \right. \right. \\
& \left. \left. - \frac{2 (b x + a) \sinh(b x + a)^2 \cosh(b x + a)}{9} - \frac{14 (b x + a) \cosh(b x + a)}{9} + \frac{2 \cosh(b x + a)^2 \sinh(b x + a)}{27} + \frac{40 \sinh(b x + a)}{27} \right) \right) \\
& - \frac{12 c^2 d^2 a \left(\frac{2 (b x + a) \sinh(b x + a)}{3} + \frac{(b x + a) \sinh(b x + a) \cosh(b x + a)^2}{3} - \frac{7 \cosh(b x + a)}{9} - \frac{\sinh(b x + a)^2 \cosh(b x + a)}{9} \right)}{b^2} \\
& + \frac{6 c^2 d^2 a^2 \left(\frac{2}{3} + \frac{\cosh(b x + a)^2}{3} \right) \sinh(b x + a)}{b^2} \\
& + \frac{4 c^3 d \left(\frac{2 (b x + a) \sinh(b x + a)}{3} + \frac{(b x + a) \sinh(b x + a) \cosh(b x + a)^2}{3} - \frac{7 \cosh(b x + a)}{9} - \frac{\sinh(b x + a)^2 \cosh(b x + a)}{9} \right)}{b} \\
& - \frac{4 c^3 d a \left(\frac{2}{3} + \frac{\cosh(b x + a)^2}{3} \right) \sinh(b x + a)}{b} + c^4 \left(\frac{2}{3} + \frac{\cosh(b x + a)^2}{3} \right) \sinh(b x + a) \Big)
\end{aligned}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int (d x + c)^3 \cosh(b x + a)^3 d x$$

Optimal (type 3, 161 leaves, 8 steps):

$$-\frac{40 d^3 \cosh(b x + a)}{9 b^4} - \frac{2 d (d x + c)^2 \cosh(b x + a)}{b^2} - \frac{2 d^3 \cosh(b x + a)^3}{27 b^4} - \frac{d (d x + c)^2 \cosh(b x + a)^3}{3 b^2} + \frac{40 d^2 (d x + c) \sinh(b x + a)}{9 b^3}$$

$$+ \frac{2(dx+c)^3 \sinh(bx+a)}{3b} + \frac{2d^2(dx+c) \cosh(bx+a)^2 \sinh(bx+a)}{9b^3} + \frac{(dx+c)^3 \cosh(bx+a)^2 \sinh(bx+a)}{3b}$$

Result (type 3, 675 leaves):

$$\begin{aligned} & \frac{1}{b} \left(\frac{1}{b^3} \left(d^3 \left(\frac{2(bx+a)^3 \sinh(bx+a)}{3} + \frac{(bx+a)^3 \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{7(bx+a)^2 \cosh(bx+a)}{3} + \frac{40(bx+a) \sinh(bx+a)}{9} \right. \right. \right. \\ & \left. \left. \left. - \frac{122 \cosh(bx+a)}{27} - \frac{(bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a)}{3} + \frac{2(bx+a) \sinh(bx+a) \cosh(bx+a)^2}{9} - \frac{2 \sinh(bx+a)^2 \cosh(bx+a)}{27} \right) \right) \\ & - \frac{1}{b^3} \left(3d^3 a \left(\frac{(bx+a)^2 \sinh(bx+a) \cosh(bx+a)^2}{3} + \frac{2(bx+a)^2 \sinh(bx+a)}{3} - \frac{2(bx+a) \sinh(bx+a)^2 \cosh(bx+a)}{9} \right. \right. \\ & \left. \left. - \frac{14(bx+a) \cosh(bx+a)}{9} + \frac{2 \cosh(bx+a)^2 \sinh(bx+a)}{27} + \frac{40 \sinh(bx+a)}{27} \right) \right) + \frac{1}{b^2} \left(3d^2 c \left(\frac{(bx+a)^2 \sinh(bx+a) \cosh(bx+a)^2}{3} \right. \right. \\ & \left. \left. + \frac{2(bx+a)^2 \sinh(bx+a)}{3} - \frac{2(bx+a) \sinh(bx+a)^2 \cosh(bx+a)}{9} - \frac{14(bx+a) \cosh(bx+a)}{9} + \frac{2 \cosh(bx+a)^2 \sinh(bx+a)}{27} \right. \right. \\ & \left. \left. + \frac{40 \sinh(bx+a)}{27} \right) \right) \\ & + \frac{3d^3 a^2 \left(\frac{2(bx+a) \sinh(bx+a)}{3} + \frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{7 \cosh(bx+a)}{9} - \frac{\sinh(bx+a)^2 \cosh(bx+a)}{9} \right)}{b^3} \\ & - \frac{6d^2 a c \left(\frac{2(bx+a) \sinh(bx+a)}{3} + \frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{7 \cosh(bx+a)}{9} - \frac{\sinh(bx+a)^2 \cosh(bx+a)}{9} \right)}{b^2} \\ & + \frac{3c^2 d \left(\frac{2(bx+a) \sinh(bx+a)}{3} + \frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{7 \cosh(bx+a)}{9} - \frac{\sinh(bx+a)^2 \cosh(bx+a)}{9} \right)}{b} \\ & - \frac{d^3 a^3 \left(\frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \sinh(bx+a)}{b^3} + \frac{3d^2 a^2 c \left(\frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \sinh(bx+a)}{b^2} - \frac{3da c^2 \left(\frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \sinh(bx+a)}{b} \\ & \left. + c^3 \left(\frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \sinh(bx+a) \right) \end{aligned}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int (dx + c)^2 \cosh(bx + a)^3 dx$$

Optimal(type 3, 111 leaves, 6 steps):

$$\begin{aligned} & -\frac{4d(dx+c)\cosh(bx+a)}{3b^2} - \frac{2d(dx+c)\cosh(bx+a)^3}{9b^2} + \frac{14d^2\sinh(bx+a)}{9b^3} + \frac{2(dx+c)^2\sinh(bx+a)}{3b} + \frac{(dx+c)^2\cosh(bx+a)^2\sinh(bx+a)}{3b} \\ & + \frac{2d^2\sinh(bx+a)^3}{27b^3} \end{aligned}$$

Result(type 3, 319 leaves):

$$\begin{aligned} & \frac{1}{b} \left(\frac{1}{b^2} \left(d^2 \left(\frac{(bx+a)^2\sinh(bx+a)\cosh(bx+a)^2}{3} + \frac{2(bx+a)^2\sinh(bx+a)}{3} - \frac{2(bx+a)\sinh(bx+a)^2\cosh(bx+a)}{9} \right. \right. \right. \\ & \left. \left. - \frac{14(bx+a)\cosh(bx+a)}{9} + \frac{2\cosh(bx+a)^2\sinh(bx+a)}{27} + \frac{40\sinh(bx+a)}{27} \right) \right) \\ & - \frac{2d^2a \left(\frac{2(bx+a)\sinh(bx+a)}{3} + \frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{3} - \frac{7\cosh(bx+a)}{9} - \frac{\sinh(bx+a)^2\cosh(bx+a)}{9} \right)}{b^2} \\ & + \frac{2dc \left(\frac{2(bx+a)\sinh(bx+a)}{3} + \frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{3} - \frac{7\cosh(bx+a)}{9} - \frac{\sinh(bx+a)^2\cosh(bx+a)}{9} \right)}{b} \\ & + \frac{d^2a^2 \left(\frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \sinh(bx+a)}{b^2} - \frac{2dac \left(\frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \sinh(bx+a)}{b} + c^2 \left(\frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \sinh(bx+a) \end{aligned}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int x^3 \cosh(bx + a)^4 dx$$

Optimal(type 3, 152 leaves, 8 steps):

$$\begin{aligned} & \frac{45x^2}{128b^2} + \frac{3x^4}{32} - \frac{45\cosh(bx+a)^2}{128b^4} - \frac{9x^2\cosh(bx+a)^2}{16b^2} - \frac{3\cosh(bx+a)^4}{128b^4} - \frac{3x^2\cosh(bx+a)^4}{16b^2} + \frac{45x\cosh(bx+a)\sinh(bx+a)}{64b^3} \\ & + \frac{3x^3\cosh(bx+a)\sinh(bx+a)}{8b} + \frac{3x\cosh(bx+a)^3\sinh(bx+a)}{32b^3} + \frac{x^3\cosh(bx+a)^3\sinh(bx+a)}{4b} \end{aligned}$$

Result(type 3, 431 leaves):

$$\frac{1}{b^4} \left(\frac{(bx+a)^3\sinh(bx+a)\cosh(bx+a)^3}{4} + \frac{3(bx+a)^3\cosh(bx+a)\sinh(bx+a)}{8} + \frac{3(bx+a)^4}{32} - \frac{3(bx+a)^2\sinh(bx+a)^2\cosh(bx+a)^2}{16} \right)$$

$$\begin{aligned}
& - \frac{3 (bx+a)^2 \cosh(bx+a)^2}{4} + \frac{3 (bx+a) \sinh(bx+a) \cosh(bx+a)^3}{32} + \frac{45 (bx+a) \cosh(bx+a) \sinh(bx+a)}{64} + \frac{45 (bx+a)^2}{128} \\
& - \frac{3 \sinh(bx+a)^2 \cosh(bx+a)^2}{128} - \frac{3 \cosh(bx+a)^2}{8} - 3a \left(\frac{(bx+a)^2 \sinh(bx+a) \cosh(bx+a)^3}{4} + \frac{3 (bx+a)^2 \cosh(bx+a) \sinh(bx+a)}{8} \right) \\
& + \frac{(bx+a)^3}{8} - \frac{(bx+a) \sinh(bx+a)^2 \cosh(bx+a)^2}{8} - \frac{(bx+a) \cosh(bx+a)^2}{2} + \frac{\cosh(bx+a)^3 \sinh(bx+a)}{32} \\
& + \frac{15 \cosh(bx+a) \sinh(bx+a)}{64} + \frac{15 bx}{64} + \frac{15 a}{64} \Big) + 3a^2 \left(\frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^3}{4} + \frac{3 (bx+a) \cosh(bx+a) \sinh(bx+a)}{8} \right) \\
& + \frac{3 (bx+a)^2}{16} - \frac{\sinh(bx+a)^2 \cosh(bx+a)^2}{16} - \frac{\cosh(bx+a)^2}{4} \Big) - a^3 \left(\left(\frac{\cosh(bx+a)^3}{4} + \frac{3 \cosh(bx+a)}{8} \right) \sinh(bx+a) + \frac{3 bx}{8} + \frac{3 a}{8} \right) \Big)
\end{aligned}$$

Problem 14: Unable to integrate problem.

$$\int (dx+c)^5 /2 \cosh(bx+a) dx$$

Optimal(type 4, 131 leaves, 8 steps):

$$\begin{aligned}
& - \frac{5 d (dx+c)^3 /2 \cosh(bx+a)}{2 b^2} + \frac{(dx+c)^5 /2 \sinh(bx+a)}{b} + \frac{15 d^5 /2 e^{-a+\frac{bc}{d}} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{\pi}}{16 b^7 /2} - \frac{15 d^5 /2 e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{\pi}}{16 b^7 /2} \\
& + \frac{15 d^2 \sinh(bx+a) \sqrt{dx+c}}{4 b^3}
\end{aligned}$$

Result(type 8, 16 leaves):

$$\int (dx+c)^5 /2 \cosh(bx+a) dx$$

Problem 15: Unable to integrate problem.

$$\int \frac{\cosh(bx+a)}{(dx+c)^3 /2} dx$$

Optimal(type 4, 91 leaves, 6 steps):

$$- \frac{e^{-a+\frac{bc}{d}} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{b} \sqrt{\pi}}{d^3 /2} + \frac{e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{b} \sqrt{\pi}}{d^3 /2} - \frac{2 \cosh(bx+a)}{d \sqrt{dx+c}}$$

Result(type 8, 16 leaves):

$$\int \frac{\cosh(bx+a)}{(dx+c)^3 /2} dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{\cosh(bx+a)}{(dx+c)^{7/2}} dx$$

Optimal(type 4, 132 leaves, 8 steps):

$$-\frac{2 \cosh(bx+a)}{5d(dx+c)^{5/2}} - \frac{4b \sinh(bx+a)}{15d^2(dx+c)^{3/2}} - \frac{4b^5/2 e^{-a+\frac{bc}{d}} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{\pi}}{15d^7/2} + \frac{4b^5/2 e^{a-\frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{\pi}}{15d^7/2} - \frac{8b^2 \cosh(bx+a)}{15d^3\sqrt{dx+c}}$$

Result(type 8, 16 leaves):

$$\int \frac{\cosh(bx+a)}{(dx+c)^{7/2}} dx$$

Problem 17: Unable to integrate problem.

$$\int (dx+c)^3/2 \cosh(bx+a)^2 dx$$

Optimal(type 4, 159 leaves, 9 steps):

$$\frac{(dx+c)^5/2}{5d} + \frac{(dx+c)^3/2 \cosh(bx+a) \sinh(bx+a)}{2b} + \frac{3d^3/2 e^{-2a+\frac{2bc}{d}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{2}\sqrt{\pi}}{128b^5/2} + \frac{3d^3/2 e^{2a-\frac{2bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{2}\sqrt{\pi}}{128b^5/2} + \frac{3d\sqrt{dx+c}}{16b^2} - \frac{3d \cosh(bx+a)^2 \sqrt{dx+c}}{8b^2}$$

Result(type 8, 18 leaves):

$$\int (dx+c)^3/2 \cosh(bx+a)^2 dx$$

Problem 18: Unable to integrate problem.

$$\int (dx+c)^5/2 \cosh(bx+a)^3 dx$$

Optimal(type 4, 291 leaves, 23 steps):

$$-\frac{5d(dx+c)^3/2 \cosh(bx+a)}{3b^2} - \frac{5d(dx+c)^3/2 \cosh(bx+a)^3}{18b^2} + \frac{2(dx+c)^5/2 \sinh(bx+a)}{3b} + \frac{(dx+c)^5/2 \cosh(bx+a)^2 \sinh(bx+a)}{3b} + \frac{5d^5/2 e^{-3a+\frac{3bc}{d}} \operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{3}\sqrt{\pi}}{1728b^7/2} - \frac{5d^5/2 e^{3a-\frac{3bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{3}\sqrt{\pi}}{1728b^7/2}$$

$$\begin{aligned}
& + \frac{45 d^5 / 2 e^{-a + \frac{bc}{d}} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{\pi}}{64 b^7 / 2} - \frac{45 d^5 / 2 e^{a - \frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{\pi}}{64 b^7 / 2} + \frac{45 d^2 \sinh(bx+a) \sqrt{dx+c}}{16 b^3} \\
& + \frac{5 d^2 \sinh(3bx+3a) \sqrt{dx+c}}{144 b^3}
\end{aligned}$$

Result(type 8, 18 leaves):

$$\int (dx+c)^{5/2} \cosh(bx+a)^3 dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{\cosh(bx+a)^3}{\sqrt{dx+c}} dx$$

Optimal(type 4, 162 leaves, 12 steps):

$$\begin{aligned}
& \frac{e^{-3a + \frac{3bc}{d}} \operatorname{erf}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{3} \sqrt{\pi}}{24 \sqrt{b} \sqrt{d}} + \frac{e^{3a - \frac{3bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{3} \sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{3} \sqrt{\pi}}{24 \sqrt{b} \sqrt{d}} + \frac{3 e^{-a + \frac{bc}{d}} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{\pi}}{8 \sqrt{b} \sqrt{d}} \\
& + \frac{3 e^{a - \frac{bc}{d}} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{dx+c}}{\sqrt{d}}\right) \sqrt{\pi}}{8 \sqrt{b} \sqrt{d}}
\end{aligned}$$

Result(type 8, 18 leaves):

$$\int \frac{\cosh(bx+a)^3}{\sqrt{dx+c}} dx$$

Problem 25: Result unnecessarily involves higher level functions.

$$\int x^{3+m} \cosh(bx+a) dx$$

Optimal(type 4, 53 leaves, 3 steps):

$$-\frac{e^a x^m \Gamma(4+m, -bx)}{2 b^4 (-bx)^m} - \frac{x^m \Gamma(4+m, bx)}{2 b^4 e^a (bx)^m}$$

Result(type 5, 72 leaves):

$$\frac{x^{4+m} \operatorname{hypergeom}\left(\left[2 + \frac{m}{2}\right], \left[\frac{1}{2}, 3 + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \cosh(a)}{4+m} + \frac{b x^{5+m} \operatorname{hypergeom}\left(\left[\frac{5}{2} + \frac{m}{2}\right], \left[\frac{3}{2}, \frac{7}{2} + \frac{m}{2}\right], \frac{x^2 b^2}{4}\right) \sinh(a)}{5+m}$$

Problem 26: Unable to integrate problem.

$$\int x^m \cosh(bx + a)^2 dx$$

Optimal(type 4, 83 leaves, 5 steps):

$$\frac{x^{1+m}}{2(1+m)} + \frac{2^{-3-m} e^{2a} x^m \Gamma(1+m, -2bx)}{b(-bx)^m} - \frac{2^{-3-m} x^m \Gamma(1+m, 2bx)}{b e^{2a} (bx)^m}$$

Result(type 8, 14 leaves):

$$\int x^m \cosh(bx + a)^2 dx$$

Problem 27: Unable to integrate problem.

$$\int x^{-1+m} \cosh(bx + a)^2 dx$$

Optimal(type 4, 70 leaves, 5 steps):

$$\frac{x^m}{2m} - \frac{2^{-2-m} e^{2a} x^m \Gamma(m, -2bx)}{(-bx)^m} - \frac{2^{-2-m} x^m \Gamma(m, 2bx)}{e^{2a} (bx)^m}$$

Result(type 8, 16 leaves):

$$\int x^{-1+m} \cosh(bx + a)^2 dx$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int (dx + c)^2 (a + a \cosh(fx + e)) dx$$

Optimal(type 3, 65 leaves, 5 steps):

$$\frac{a(dx+c)^3}{3d} - \frac{2ad(dx+c)\cosh(fx+e)}{f^2} + \frac{2ad^2\sinh(fx+e)}{f^3} + \frac{a(dx+c)^2\sinh(fx+e)}{f}$$

Result(type 3, 239 leaves):

$$\frac{1}{f} \left(\frac{d^2 a (fx+e)^3}{3f^2} + \frac{d^2 a ((fx+e)^2 \sinh(fx+e) - 2(fx+e) \cosh(fx+e) + 2 \sinh(fx+e))}{f^2} - \frac{d^2 e a (fx+e)^2}{f^2} \right. \\ \left. - \frac{2d^2 e a ((fx+e) \sinh(fx+e) - \cosh(fx+e))}{f^2} + \frac{dca (fx+e)^2}{f} + \frac{2dca ((fx+e) \sinh(fx+e) - \cosh(fx+e))}{f} + \frac{d^2 e^2 a (fx+e)}{f^2} \right. \\ \left. + \frac{d^2 e^2 a \sinh(fx+e)}{f^2} - \frac{2deca (fx+e)}{f} - \frac{2deca \sinh(fx+e)}{f} + c^2 a (fx+e) + c^2 a \sinh(fx+e) \right)$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int (dx + c) (a + a \cosh(fx + e)) dx$$

Optimal(type 3, 43 leaves, 4 steps):

$$\frac{a(dx+c)^2}{2d} - \frac{ad \cosh(fx+e)}{f^2} + \frac{a(dx+c) \sinh(fx+e)}{f}$$

Result(type 3, 90 leaves):

$$\frac{\frac{da(fx+e)^2}{2f} + \frac{da((fx+e) \sinh(fx+e) - \cosh(fx+e))}{f} - \frac{dea(fx+e)}{f} - \frac{dea \sinh(fx+e)}{f} + ca(fx+e) + ca \sinh(fx+e)}{f}$$

Problem 37: Unable to integrate problem.

$$\int \frac{\sqrt{a+a \cosh(dx+c)}}{x} dx$$

Optimal(type 4, 63 leaves, 4 steps):

$$\text{Chi}\left(\frac{dx}{2}\right) \cosh\left(\frac{c}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a+a \cosh(dx+c)} + \text{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \text{Shi}\left(\frac{dx}{2}\right) \sinh\left(\frac{c}{2}\right) \sqrt{a+a \cosh(dx+c)}$$

Result(type 8, 18 leaves):

$$\int \frac{\sqrt{a+a \cosh(dx+c)}}{x} dx$$

Problem 38: Unable to integrate problem.

$$\int \frac{\sqrt{a+a \cosh(dx+c)}}{x^3} dx$$

Optimal(type 4, 115 leaves, 6 steps):

$$-\frac{\sqrt{a+a \cosh(dx+c)}}{2x^2} + \frac{d^2 \text{Chi}\left(\frac{dx}{2}\right) \cosh\left(\frac{c}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a+a \cosh(dx+c)}}{8} + \frac{d^2 \text{sech}\left(\frac{c}{2} + \frac{dx}{2}\right) \text{Shi}\left(\frac{dx}{2}\right) \sinh\left(\frac{c}{2}\right) \sqrt{a+a \cosh(dx+c)}}{8} - \frac{d \sqrt{a+a \cosh(dx+c)} \tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{4x}$$

Result(type 8, 18 leaves):

$$\int \frac{\sqrt{a+a \cosh(dx+c)}}{x^3} dx$$

Problem 39: Unable to integrate problem.

$$\int \frac{\sqrt{a+a \cosh(x)}}{x^2} dx$$

Optimal(type 4, 32 leaves, 3 steps):

$$-\frac{\sqrt{a+a\cosh(x)}}{x} + \frac{\operatorname{sech}\left(\frac{x}{2}\right) \operatorname{Shi}\left(\frac{x}{2}\right) \sqrt{a+a\cosh(x)}}{2}$$

Result(type 8, 14 leaves):

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2} dx$$

Problem 40: Unable to integrate problem.

$$\int x^3 (a+a\cosh(x))^3 /2 dx$$

Optimal(type 3, 139 leaves, 9 steps):

$$\begin{aligned} & -\frac{1280 a \sqrt{a+a\cosh(x)}}{9} - 16 a x^2 \sqrt{a+a\cosh(x)} - \frac{64 a \cosh\left(\frac{x}{2}\right)^2 \sqrt{a+a\cosh(x)}}{27} - \frac{8 a x^2 \cosh\left(\frac{x}{2}\right)^2 \sqrt{a+a\cosh(x)}}{3} \\ & + \frac{32 a x \cosh\left(\frac{x}{2}\right) \sinh\left(\frac{x}{2}\right) \sqrt{a+a\cosh(x)}}{9} + \frac{4 a x^3 \cosh\left(\frac{x}{2}\right) \sinh\left(\frac{x}{2}\right) \sqrt{a+a\cosh(x)}}{3} + \frac{640 a x \sqrt{a+a\cosh(x)} \tanh\left(\frac{x}{2}\right)}{9} \\ & + \frac{8 a x^3 \sqrt{a+a\cosh(x)} \tanh\left(\frac{x}{2}\right)}{3} \end{aligned}$$

Result(type 8, 14 leaves):

$$\int x^3 (a+a\cosh(x))^3 /2 dx$$

Problem 44: Unable to integrate problem.

$$\int (dx+c)^m (a+a\cosh(fx+e))^2 dx$$

Optimal(type 4, 257 leaves, 9 steps):

$$\begin{aligned} & \frac{3 a^2 (dx+c)^{1+m}}{2 d (1+m)} + \frac{2^{-3-m} a^2 e^{2e-\frac{2cf}{d}} (dx+c)^m \Gamma\left(1+m, -\frac{2f(dx+c)}{d}\right)}{f\left(-\frac{f(dx+c)}{d}\right)^m} + \frac{a^2 e^{-\frac{cf}{d}} (dx+c)^m \Gamma\left(1+m, -\frac{f(dx+c)}{d}\right)}{f\left(-\frac{f(dx+c)}{d}\right)^m} \\ & - \frac{a^2 e^{-e+\frac{cf}{d}} (dx+c)^m \Gamma\left(1+m, \frac{f(dx+c)}{d}\right)}{f\left(\frac{f(dx+c)}{d}\right)^m} - \frac{2^{-3-m} a^2 e^{-2e+\frac{2cf}{d}} (dx+c)^m \Gamma\left(1+m, \frac{2f(dx+c)}{d}\right)}{f\left(\frac{f(dx+c)}{d}\right)^m} \end{aligned}$$

Result(type 8, 22 leaves):

$$\int (dx + c)^m (a + a \cosh(fx + e))^2 dx$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int (dx + c)^3 (a + b \cosh(fx + e)) dx$$

Optimal (type 3, 87 leaves, 6 steps):

$$\frac{a(dx+c)^4}{4d} - \frac{6bd^3 \cosh(fx+e)}{f^4} - \frac{3bd(dx+c)^2 \cosh(fx+e)}{f^2} + \frac{6bd^2(dx+c) \sinh(fx+e)}{f^3} + \frac{b(dx+c)^3 \sinh(fx+e)}{f}$$

Result (type 3, 481 leaves):

$$\begin{aligned} & \frac{1}{f} \left(\frac{d^3 a (fx+e)^4}{4f^3} + \frac{d^3 b ((fx+e)^3 \sinh(fx+e) - 3(fx+e)^2 \cosh(fx+e) + 6(fx+e) \sinh(fx+e) - 6 \cosh(fx+e))}{f^3} - \frac{d^3 e a (fx+e)^3}{f^3} \right. \\ & - \frac{3d^3 e b ((fx+e)^2 \sinh(fx+e) - 2(fx+e) \cosh(fx+e) + 2 \sinh(fx+e))}{f^3} + \frac{d^2 c a (fx+e)^3}{f^2} \\ & + \frac{3d^2 c b ((fx+e)^2 \sinh(fx+e) - 2(fx+e) \cosh(fx+e) + 2 \sinh(fx+e))}{f^2} + \frac{3d^3 e^2 a (fx+e)^2}{2f^3} \\ & + \frac{3d^3 e^2 b ((fx+e) \sinh(fx+e) - \cosh(fx+e))}{f^3} - \frac{3d^2 e c a (fx+e)^2}{f^2} - \frac{6d^2 e c b ((fx+e) \sinh(fx+e) - \cosh(fx+e))}{f^2} + \frac{3d c^2 a (fx+e)^2}{2f} \\ & + \frac{3d c^2 b ((fx+e) \sinh(fx+e) - \cosh(fx+e))}{f} - \frac{d^3 e^3 a (fx+e)}{f^3} - \frac{d^3 e^3 b \sinh(fx+e)}{f^3} + \frac{3d^2 e^2 c a (fx+e)}{f^2} + \frac{3d^2 e^2 c b \sinh(fx+e)}{f^2} \\ & \left. - \frac{3d e c^2 a (fx+e)}{f} - \frac{3d e c^2 b \sinh(fx+e)}{f} + a c^3 (fx+e) + c^3 b \sinh(fx+e) \right) \end{aligned}$$

Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \cosh(fx + e))^2}{(dx + c)^3} dx$$

Optimal (type 4, 242 leaves, 14 steps):

$$\begin{aligned} & -\frac{a^2}{2d(dx+c)^2} + \frac{b^2 f^2 \operatorname{Chi}\left(\frac{2cf}{d} + 2fx\right) \cosh\left(-2e + \frac{2cf}{d}\right)}{d^3} + \frac{abf^2 \operatorname{Chi}\left(\frac{cf}{d} + fx\right) \cosh\left(-e + \frac{cf}{d}\right)}{d^3} - \frac{ab \cosh(fx+e)}{d(dx+c)^2} - \frac{b^2 \cosh(fx+e)^2}{2d(dx+c)^2} \\ & - \frac{b^2 f^2 \operatorname{Shi}\left(\frac{2cf}{d} + 2fx\right) \sinh\left(-2e + \frac{2cf}{d}\right)}{d^3} - \frac{abf^2 \operatorname{Shi}\left(\frac{cf}{d} + fx\right) \sinh\left(-e + \frac{cf}{d}\right)}{d^3} - \frac{abf \sinh(fx+e)}{d^2(dx+c)} - \frac{b^2 f \cosh(fx+e) \sinh(fx+e)}{d^2(dx+c)} \end{aligned}$$

Result (type 4, 625 leaves):

$$\begin{aligned}
& \frac{a b f^3 e^{-fx-e} x}{2 d (d^2 f^2 x^2 + 2 c d f^2 x + c^2 f^2)} + \frac{a b f^3 e^{-fx-e} c}{2 d^2 (d^2 f^2 x^2 + 2 c d f^2 x + c^2 f^2)} - \frac{a b f^2 e^{-fx-e}}{2 d (d^2 f^2 x^2 + 2 c d f^2 x + c^2 f^2)} - \frac{a b f^2 e^{-\frac{cf-de}{d}} \operatorname{Ei}_1\left(fx + e + \frac{cf-de}{d}\right)}{2 d^3} \\
& - \frac{a b f^2 e^{fx+e}}{2 d^3 \left(\frac{cf}{d} + fx\right)^2} - \frac{a b f^2 e^{fx+e}}{2 d^3 \left(\frac{cf}{d} + fx\right)} - \frac{a b f^2 e^{-\frac{cf-de}{d}} \operatorname{Ei}_1\left(-fx - e - \frac{cf-de}{d}\right)}{2 d^3} - \frac{a^2}{2 d (dx + c)^2} - \frac{b^2}{4 d (dx + c)^2} \\
& + \frac{b^2 f^3 e^{-2fx-2e} x}{4 d (d^2 f^2 x^2 + 2 c d f^2 x + c^2 f^2)} + \frac{b^2 f^3 e^{-2fx-2e} c}{4 d^2 (d^2 f^2 x^2 + 2 c d f^2 x + c^2 f^2)} - \frac{b^2 f^2 e^{-2fx-2e}}{8 d (d^2 f^2 x^2 + 2 c d f^2 x + c^2 f^2)} \\
& - \frac{b^2 f^2 e^{\frac{2(cf-de)}{d}} \operatorname{Ei}_1\left(2fx + 2e + \frac{2(cf-de)}{d}\right)}{2 d^3} - \frac{f^2 b^2 e^{2fx+2e}}{8 d^3 \left(\frac{cf}{d} + fx\right)^2} - \frac{f^2 b^2 e^{2fx+2e}}{4 d^3 \left(\frac{cf}{d} + fx\right)} - \frac{f^2 b^2 e^{-\frac{2(cf-de)}{d}} \operatorname{Ei}_1\left(-2fx - 2e - \frac{2(cf-de)}{d}\right)}{2 d^3}
\end{aligned}$$

Test results for the 30 problems in "6.2.2 (e x)^m (a+b x^n)^p cosh.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int x^3 (bx + a) \cosh(dx + c) dx$$

Optimal (type 3, 124 leaves, 11 steps):

$$\begin{aligned}
& -\frac{6 a \cosh(dx + c)}{d^4} - \frac{24 b x \cosh(dx + c)}{d^4} - \frac{3 a x^2 \cosh(dx + c)}{d^2} - \frac{4 b x^3 \cosh(dx + c)}{d^2} + \frac{24 b \sinh(dx + c)}{d^5} + \frac{6 a x \sinh(dx + c)}{d^3} + \frac{12 b x^2 \sinh(dx + c)}{d^3} \\
& + \frac{a x^3 \sinh(dx + c)}{d} + \frac{b x^4 \sinh(dx + c)}{d}
\end{aligned}$$

Result (type 3, 355 leaves):

$$\begin{aligned}
& \frac{1}{d^4} \left(\frac{b \left((dx + c)^4 \sinh(dx + c) - 4 (dx + c)^3 \cosh(dx + c) + 12 (dx + c)^2 \sinh(dx + c) - 24 (dx + c) \cosh(dx + c) + 24 \sinh(dx + c) \right)}{d} \right. \\
& - \frac{4 b c \left((dx + c)^3 \sinh(dx + c) - 3 (dx + c)^2 \cosh(dx + c) + 6 (dx + c) \sinh(dx + c) - 6 \cosh(dx + c) \right)}{d} \\
& + \frac{6 b c^2 \left((dx + c)^2 \sinh(dx + c) - 2 (dx + c) \cosh(dx + c) + 2 \sinh(dx + c) \right)}{d} - \frac{4 b c^3 \left((dx + c) \sinh(dx + c) - \cosh(dx + c) \right)}{d} \\
& \left. + \frac{b c^4 \sinh(dx + c)}{d} + a \left((dx + c)^3 \sinh(dx + c) - 3 (dx + c)^2 \cosh(dx + c) + 6 (dx + c) \sinh(dx + c) - 6 \cosh(dx + c) \right) - 3 a c \left((dx + c)^2 \sinh(dx + c) - 2 (dx + c) \cosh(dx + c) + 2 \sinh(dx + c) \right) + 3 a c^2 \left((dx + c) \sinh(dx + c) - \cosh(dx + c) \right) - a c^3 \sinh(dx + c) \right)
\end{aligned}$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int x^2 (bx+a)^2 \cosh(dx+c) dx$$

Optimal (type 3, 184 leaves, 14 steps):

$$\begin{aligned} & -\frac{12ab \cosh(dx+c)}{d^4} - \frac{24b^2x \cosh(dx+c)}{d^4} - \frac{2a^2x \cosh(dx+c)}{d^2} - \frac{6abx^2 \cosh(dx+c)}{d^2} - \frac{4b^2x^3 \cosh(dx+c)}{d^2} + \frac{24b^2 \sinh(dx+c)}{d^5} \\ & + \frac{2a^2 \sinh(dx+c)}{d^3} + \frac{12abx \sinh(dx+c)}{d^3} + \frac{12b^2x^2 \sinh(dx+c)}{d^3} + \frac{a^2x^2 \sinh(dx+c)}{d} + \frac{2abx^3 \sinh(dx+c)}{d} + \frac{b^2x^4 \sinh(dx+c)}{d} \end{aligned}$$

Result (type 3, 462 leaves):

$$\begin{aligned} & \frac{1}{d^3} \left(\frac{b^2((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c))}{d^2} \right. \\ & - \frac{4b^2c((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d^2} \\ & + \frac{6b^2c^2((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d^2} - \frac{4b^2c^3((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d^2} \\ & + \frac{2ab((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d} \\ & - \frac{6bca((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d} + \frac{6ba^2c((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d} \\ & + \frac{b^2c^4 \sinh(dx+c)}{d^2} - \frac{2b^3a \sinh(dx+c)}{d} + a^2((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c)) - 2a^2c((dx+c) \sinh(dx+c) \\ & \left. + c) - \cosh(dx+c) + a^2c^2 \sinh(dx+c) \right) \end{aligned}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{x \cosh(dx+c)}{(bx+a)^3} dx$$

Optimal (type 4, 175 leaves, 11 steps):

$$\begin{aligned} & -\frac{ad^2 \operatorname{Chi}\left(\frac{da}{b} + dx\right) \cosh\left(-c + \frac{da}{b}\right)}{2b^4} + \frac{a \cosh(dx+c)}{2b^2(bx+a)^2} - \frac{\cosh(dx+c)}{b^2(bx+a)} + \frac{d \cosh\left(-c + \frac{da}{b}\right) \operatorname{Shi}\left(\frac{da}{b} + dx\right)}{b^3} - \frac{d \operatorname{Chi}\left(\frac{da}{b} + dx\right) \sinh\left(-c + \frac{da}{b}\right)}{b^3} \\ & + \frac{ad^2 \operatorname{Shi}\left(\frac{da}{b} + dx\right) \sinh\left(-c + \frac{da}{b}\right)}{2b^4} + \frac{ad \sinh(dx+c)}{2b^3(bx+a)} \end{aligned}$$

Result (type 4, 434 leaves):

$$-\frac{d^3 e^{-dx-c} ax}{4b^2(b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^3 e^{-dx-c} a^2}{4b^3(b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c} x}{2b(b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c} a}{4b^2(b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)}$$

$$\begin{aligned}
& + \frac{d^2 e^{\frac{da-cb}{b}} \operatorname{Ei}_1\left(dx+c+\frac{da-cb}{b}\right) a}{4b^4} + \frac{d e^{\frac{da-cb}{b}} \operatorname{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{2b^3} + \frac{d^2 e^{dx+c} a}{4b^4 \left(\frac{da}{b}+dx\right)^2} + \frac{d^2 e^{dx+c} a}{4b^4 \left(\frac{da}{b}+dx\right)} \\
& + \frac{d^2 e^{-\frac{da-cb}{b}} \operatorname{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right) a}{4b^4} - \frac{d e^{dx+c}}{2b^3 \left(\frac{da}{b}+dx\right)} - \frac{d e^{-\frac{da-cb}{b}} \operatorname{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)}{2b^3}
\end{aligned}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(dx+c)}{x^3 (bx+a)^3} dx$$

Optimal (type 4, 367 leaves, 26 steps):

$$\begin{aligned}
& \frac{6b^2 \operatorname{Chi}(dx) \cosh(c)}{a^5} + \frac{d^2 \operatorname{Chi}(dx) \cosh(c)}{2a^3} - \frac{6b^2 \operatorname{Chi}\left(\frac{da}{b}+dx\right) \cosh\left(-c+\frac{da}{b}\right)}{a^5} - \frac{d^2 \operatorname{Chi}\left(\frac{da}{b}+dx\right) \cosh\left(-c+\frac{da}{b}\right)}{2a^3} - \frac{\cosh(dx+c)}{2a^3 x^2} \\
& + \frac{3b \cosh(dx+c)}{a^4 x} + \frac{b^2 \cosh(dx+c)}{2a^3 (bx+a)^2} + \frac{3b^2 \cosh(dx+c)}{a^4 (bx+a)} - \frac{3bd \cosh(c) \operatorname{Shi}(dx)}{a^4} - \frac{3bd \cosh\left(-c+\frac{da}{b}\right) \operatorname{Shi}\left(\frac{da}{b}+dx\right)}{a^4} \\
& - \frac{3bd \operatorname{Chi}(dx) \sinh(c)}{a^4} + \frac{6b^2 \operatorname{Shi}(dx) \sinh(c)}{a^5} + \frac{d^2 \operatorname{Shi}(dx) \sinh(c)}{2a^3} + \frac{3bd \operatorname{Chi}\left(\frac{da}{b}+dx\right) \sinh\left(-c+\frac{da}{b}\right)}{a^4} \\
& + \frac{6b^2 \operatorname{Shi}\left(\frac{da}{b}+dx\right) \sinh\left(-c+\frac{da}{b}\right)}{a^5} + \frac{d^2 \operatorname{Shi}\left(\frac{da}{b}+dx\right) \sinh\left(-c+\frac{da}{b}\right)}{2a^3} - \frac{d \sinh(dx+c)}{2a^3 x} + \frac{bd \sinh(dx+c)}{2a^3 (bx+a)}
\end{aligned}$$

Result (type 4, 759 leaves):

$$\begin{aligned}
& \frac{d^3 e^{-dx-cb}}{4a^2 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{3d^2 e^{-dx-c} x b^3}{a^4 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{d^3 e^{-dx-c}}{4ax (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{9d^2 e^{-dx-c} b^2}{2a^3 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} \\
& + \frac{d^2 e^{-dx-cb}}{a^2 x (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c}}{4ax^2 (b^2 d^2 x^2 + 2ab d^2 x + a^2 d^2)} + \frac{d^2 e^{\frac{da-cb}{b}} \operatorname{Ei}_1\left(dx+c+\frac{da-cb}{b}\right)}{4a^3} \\
& - \frac{3d e^{\frac{da-cb}{b}} \operatorname{Ei}_1\left(dx+c+\frac{da-cb}{b}\right) b}{2a^4} + \frac{3e^{\frac{da-cb}{b}} \operatorname{Ei}_1\left(dx+c+\frac{da-cb}{b}\right) b^2}{a^5} - \frac{d^2 e^{-c} \operatorname{Ei}_1(dx)}{4a^3} - \frac{3d e^{-c} \operatorname{Ei}_1(dx) b}{2a^4} - \frac{3e^{-c} \operatorname{Ei}_1(dx) b^2}{a^5}
\end{aligned}$$

$$\begin{aligned}
& + \frac{d^2 e^{dx+c}}{4a^3 \left(\frac{da}{b} + dx\right)^2} + \frac{d^2 e^{dx+c}}{4a^3 \left(\frac{da}{b} + dx\right)} + \frac{d^2 e^{-\frac{da-cb}{b}} \operatorname{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)}{4a^3} + \frac{3db e^{dx+c}}{2a^4 \left(\frac{da}{b} + dx\right)} \\
& + \frac{3db e^{-\frac{da-cb}{b}} \operatorname{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)}{2a^4} + \frac{3b e^{dx+c}}{2a^4 x} + \frac{3db e^c \operatorname{Ei}_1(-dx)}{2a^4} + \frac{3b^2 e^{-\frac{da-cb}{b}} \operatorname{Ei}_1\left(-dx-c-\frac{da-cb}{b}\right)}{a^5} - \frac{3b^2 e^c \operatorname{Ei}_1(-dx)}{a^5} \\
& - \frac{e^{dx+c}}{4a^3 x^2} - \frac{d e^{dx+c}}{4a^3 x} - \frac{d^2 e^c \operatorname{Ei}_1(-dx)}{4a^3}
\end{aligned}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int x^2 (bx^2 + a) \cosh(dx + c) dx$$

Optimal (type 3, 109 leaves, 10 steps):

$$\begin{aligned}
& - \frac{24bx \cosh(dx+c)}{d^4} - \frac{2ax \cosh(dx+c)}{d^2} - \frac{4bx^3 \cosh(dx+c)}{d^2} + \frac{24b \sinh(dx+c)}{d^5} + \frac{2a \sinh(dx+c)}{d^3} + \frac{12bx^2 \sinh(dx+c)}{d^3} + \frac{ax^2 \sinh(dx+c)}{d} \\
& + \frac{bx^4 \sinh(dx+c)}{d}
\end{aligned}$$

Result (type 3, 297 leaves):

$$\begin{aligned}
& \frac{1}{d^3} \left(\frac{b \left((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c) \right)}{d^2} \right. \\
& - \frac{4bc \left((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c) \right)}{d^2} \\
& + \frac{6b^2 \left((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c) \right)}{d^2} - \frac{4b^3 \left((dx+c) \sinh(dx+c) - \cosh(dx+c) \right)}{d^2} \\
& + \frac{bc^4 \sinh(dx+c)}{d^2} + a \left((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c) \right) - 2ac \left((dx+c) \sinh(dx+c) - \cosh(dx+c) \right) \\
& \left. + c^2 a \sinh(dx+c) \right)
\end{aligned}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int x^2 (bx^2 + a)^2 \cosh(dx + c) dx$$

Optimal (type 3, 234 leaves, 17 steps):

$$- \frac{720b^2 x \cosh(dx+c)}{d^6} - \frac{48abx \cosh(dx+c)}{d^4} - \frac{2a^2 x \cosh(dx+c)}{d^2} - \frac{120b^2 x^3 \cosh(dx+c)}{d^4} - \frac{8abx^3 \cosh(dx+c)}{d^2} - \frac{6b^2 x^5 \cosh(dx+c)}{d^2}$$

$$\begin{aligned}
& + \frac{720 b^2 \sinh(dx+c)}{d^7} + \frac{48 a b \sinh(dx+c)}{d^5} + \frac{2 a^2 \sinh(dx+c)}{d^3} + \frac{360 b^2 x^2 \sinh(dx+c)}{d^5} + \frac{24 a b x^2 \sinh(dx+c)}{d^3} + \frac{a^2 x^2 \sinh(dx+c)}{d} \\
& + \frac{30 b^2 x^4 \sinh(dx+c)}{d^3} + \frac{2 a b x^4 \sinh(dx+c)}{d} + \frac{b^2 x^6 \sinh(dx+c)}{d}
\end{aligned}$$

Result (type 3, 737 leaves):

$$\begin{aligned}
& \frac{1}{d^3} \left(a^2 c^2 \sinh(dx+c) - \frac{8 b a c \left((dx+c)^3 \sinh(dx+c) - 3 (dx+c)^2 \cosh(dx+c) + 6 (dx+c) \sinh(dx+c) - 6 \cosh(dx+c) \right)}{d^2} \right) \\
& + \frac{12 b c^2 a \left((dx+c)^2 \sinh(dx+c) - 2 (dx+c) \cosh(dx+c) + 2 \sinh(dx+c) \right)}{d^2} - \frac{8 b c^3 a \left((dx+c) \sinh(dx+c) - \cosh(dx+c) \right)}{d^2} \\
& - \frac{1}{d^4} \left(6 b^2 c \left((dx+c)^5 \sinh(dx+c) - 5 (dx+c)^4 \cosh(dx+c) + 20 (dx+c)^3 \sinh(dx+c) - 60 (dx+c)^2 \cosh(dx+c) + 120 (dx+c) \sinh(dx+c) \right) \right. \\
& \left. + c \right) - 120 \cosh(dx+c) \left. \right) \\
& + \frac{15 b^2 c^2 \left((dx+c)^4 \sinh(dx+c) - 4 (dx+c)^3 \cosh(dx+c) + 12 (dx+c)^2 \sinh(dx+c) - 24 (dx+c) \cosh(dx+c) + 24 \sinh(dx+c) \right)}{d^4} \\
& - \frac{20 b^2 c^3 \left((dx+c)^3 \sinh(dx+c) - 3 (dx+c)^2 \cosh(dx+c) + 6 (dx+c) \sinh(dx+c) - 6 \cosh(dx+c) \right)}{d^4} \\
& + \frac{15 b^2 c^4 \left((dx+c)^2 \sinh(dx+c) - 2 (dx+c) \cosh(dx+c) + 2 \sinh(dx+c) \right)}{d^4} \\
& + \frac{2 b a \left((dx+c)^4 \sinh(dx+c) - 4 (dx+c)^3 \cosh(dx+c) + 12 (dx+c)^2 \sinh(dx+c) - 24 (dx+c) \cosh(dx+c) + 24 \sinh(dx+c) \right)}{d^2} \\
& - \frac{6 b^2 c^5 \left((dx+c) \sinh(dx+c) - \cosh(dx+c) \right)}{d^4} + \frac{2 b c^4 a \sinh(dx+c)}{d^2} + a^2 \left((dx+c)^2 \sinh(dx+c) - 2 (dx+c) \cosh(dx+c) + 2 \sinh(dx+c) \right) \\
& + c \left. \right) + \frac{1}{d^4} \left(b^2 \left((dx+c)^6 \sinh(dx+c) - 6 (dx+c)^5 \cosh(dx+c) + 30 (dx+c)^4 \sinh(dx+c) - 120 (dx+c)^3 \cosh(dx+c) + 360 (dx+c)^2 \sinh(dx+c) \right. \right. \\
& \left. \left. - 720 (dx+c) \cosh(dx+c) + 720 \sinh(dx+c) \right) \right) + \frac{b^2 c^6 \sinh(dx+c)}{d^4} - 2 a^2 c \left((dx+c) \sinh(dx+c) - \cosh(dx+c) \right) \left. \right)
\end{aligned}$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int x^2 (b x^3 + a) \cosh(dx+c) dx$$

Optimal (type 3, 124 leaves, 11 steps):

$$\begin{aligned}
& - \frac{120 b \cosh(dx+c)}{d^6} - \frac{2 a x \cosh(dx+c)}{d^2} - \frac{60 b x^2 \cosh(dx+c)}{d^4} - \frac{5 b x^4 \cosh(dx+c)}{d^2} + \frac{2 a \sinh(dx+c)}{d^3} + \frac{120 b x \sinh(dx+c)}{d^5} \\
& + \frac{a x^2 \sinh(dx+c)}{d} + \frac{20 b x^3 \sinh(dx+c)}{d^3} + \frac{b x^5 \sinh(dx+c)}{d}
\end{aligned}$$

Result(type 3, 388 leaves):

$$\begin{aligned} & \frac{1}{d^3} \left(\frac{1}{d^3} (b((dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) \right. \\ & \quad \left. - 120 \cosh(dx+c)) \right) \\ & \quad - \frac{5bc((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c))}{d^3} \\ & \quad + \frac{10b^2c^2((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d^3} \\ & \quad - \frac{10bc^3((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d^3} + \frac{5bc^4((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d^3} \\ & \quad - \frac{bc^5 \sinh(dx+c)}{d^3} + a((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c)) - 2ac((dx+c) \sinh(dx+c) - \cosh(dx+c)) \\ & \quad \left. + c^2 a \sinh(dx+c) \right) \end{aligned}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x(bx^3 + a) \cosh(dx+c) dx$$

Optimal(type 3, 94 leaves, 9 steps):

$$-\frac{a \cosh(dx+c)}{d^2} - \frac{24bx \cosh(dx+c)}{d^4} - \frac{4bx^3 \cosh(dx+c)}{d^2} + \frac{24b \sinh(dx+c)}{d^5} + \frac{ax \sinh(dx+c)}{d} + \frac{12bx^2 \sinh(dx+c)}{d^3} + \frac{bx^4 \sinh(dx+c)}{d}$$

Result(type 3, 256 leaves):

$$\begin{aligned} & \frac{1}{d^2} \left(\frac{b((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c))}{d^3} \right. \\ & \quad \left. - \frac{4bc((dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c))}{d^3} \right) \\ & \quad + \frac{6b^2c^2((dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c))}{d^3} - \frac{4bc^3((dx+c) \sinh(dx+c) - \cosh(dx+c))}{d^3} + a((dx+c) \sinh(dx+c) - \cosh(dx+c)) \\ & \quad \left. + \frac{bc^4 \sinh(dx+c)}{d^3} - ca \sinh(dx+c) \right) \end{aligned}$$

Problem 28: Result is not expressed in closed-form.

$$\int \frac{\cosh(dx+c)}{x^2(bx^3+a)} dx$$

Optimal(type 4, 273 leaves, 17 steps):

$$\begin{aligned}
& \frac{b^{1/3} \operatorname{Chi}\left(\frac{a^{1/3}d}{b^{1/3}} + dx\right) \cosh\left(c - \frac{a^{1/3}d}{b^{1/3}}\right)}{3a^{4/3}} + \frac{(-1)^{2/3} b^{1/3} \operatorname{Chi}\left(\frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}} - dx\right) \cosh\left(c + \frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}}\right)}{3a^{4/3}} \\
& - \frac{(-1)^{1/3} b^{1/3} \operatorname{Chi}\left(-\frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}} - dx\right) \cosh\left(c - \frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}}\right)}{3a^{4/3}} - \frac{\cosh(dx+c)}{ax} + \frac{d \cosh(c) \operatorname{Shi}(dx)}{a} + \frac{d \operatorname{Chi}(dx) \sinh(c)}{a} \\
& + \frac{b^{1/3} \operatorname{Shi}\left(\frac{a^{1/3}d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{a^{1/3}d}{b^{1/3}}\right)}{3a^{4/3}} + \frac{(-1)^{2/3} b^{1/3} \operatorname{Shi}\left(-\frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}} + dx\right) \sinh\left(c + \frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}}\right)}{3a^{4/3}} \\
& - \frac{(-1)^{1/3} b^{1/3} \operatorname{Shi}\left(\frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}}\right)}{3a^{4/3}}
\end{aligned}$$

Result(type 7, 186 leaves):

$$\begin{aligned}
& -\frac{e^{-dx-c}}{2xa} + \frac{d \left(\sum_{Rl=RootOf(Z^3b-3Z^2bc+3Zbc^2+ad^3-bc^3)} \frac{e^{-Rl} \operatorname{Ei}_1(dx - Rl + c)}{-Rl - c} \right)}{6a} + \frac{d e^{-c} \operatorname{Ei}_1(dx)}{2a} - \frac{e^{dx+c}}{2xa} \\
& + \frac{d \left(\sum_{Rl=RootOf(Z^3b-3Z^2bc+3Zbc^2+ad^3-bc^3)} \frac{e^{Rl} \operatorname{Ei}_1(-dx + Rl - c)}{-Rl - c} \right)}{6a} - \frac{d e^c \operatorname{Ei}_1(-dx)}{2a}
\end{aligned}$$

Problem 29: Result is not expressed in closed-form.

$$\int \frac{\cosh(dx+c)}{x^3(bx^3+a)} dx$$

Optimal(type 4, 294 leaves, 18 steps):

$$\begin{aligned}
& \frac{d^2 \operatorname{Chi}(dx) \cosh(c)}{2a} - \frac{b^{2/3} \operatorname{Chi}\left(\frac{a^{1/3}d}{b^{1/3}} + dx\right) \cosh\left(c - \frac{a^{1/3}d}{b^{1/3}}\right)}{3a^{5/3}} + \frac{(-1)^{1/3} b^{2/3} \operatorname{Chi}\left(\frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}} - dx\right) \cosh\left(c + \frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}}\right)}{3a^{5/3}} \\
& - \frac{(-1)^{2/3} b^{2/3} \operatorname{Chi}\left(-\frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}} - dx\right) \cosh\left(c - \frac{(-1)^{2/3} a^{1/3}d}{b^{1/3}}\right)}{3a^{5/3}} - \frac{\cosh(dx+c)}{2ax^2} + \frac{d^2 \operatorname{Shi}(dx) \sinh(c)}{2a} \\
& - \frac{b^{2/3} \operatorname{Shi}\left(\frac{a^{1/3}d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{a^{1/3}d}{b^{1/3}}\right)}{3a^{5/3}} + \frac{(-1)^{1/3} b^{2/3} \operatorname{Shi}\left(-\frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}} + dx\right) \sinh\left(c + \frac{(-1)^{1/3} a^{1/3}d}{b^{1/3}}\right)}{3a^{5/3}}
\end{aligned}$$

$$-\frac{(-1)^{2/3} b^{2/3} \operatorname{Shi}\left(\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right)}{3 a^{5/3}} - \frac{d \sinh(dx + c)}{2 a x}$$

Result(type 7, 239 leaves):

$$\begin{aligned} & \frac{d e^{-dx-c}}{4 x a} - \frac{e^{-dx-c}}{4 x^2 a} + \frac{d^2 \left(\sum_{Rl=RootOf(Z^3 b-3 Z^2 b c+3 Z b c^2+a d^3-b c^3)} \frac{e^{-Rl} \operatorname{Ei}_1(dx - Rl + c)}{Rl^2 - 2 Rl c + c^2} \right)}{6 a} - \frac{d^2 e^{-c} \operatorname{Ei}_1(dx)}{4 a} - \frac{d e^{dx+c}}{4 x a} - \frac{e^{dx+c}}{4 x^2 a} \\ & + \frac{d^2 \left(\sum_{Rl=RootOf(Z^3 b-3 Z^2 b c+3 Z b c^2+a d^3-b c^3)} \frac{e^{Rl} \operatorname{Ei}_1(-dx + Rl - c)}{Rl^2 - 2 Rl c + c^2} \right)}{6 a} - \frac{d^2 e^c \operatorname{Ei}_1(-dx)}{4 a} \end{aligned}$$

Problem 30: Result is not expressed in closed-form.

$$\int \frac{x^3 \cosh(dx + c)}{(b x^3 + a)^2} dx$$

Optimal(type 4, 500 leaves, 23 steps):

$$\begin{aligned} & \frac{\operatorname{Chi}\left(\frac{a^{1/3} d}{b^{1/3}} + dx\right) \cosh\left(c - \frac{a^{1/3} d}{b^{1/3}}\right)}{9 a^2/3 b^4/3} - \frac{(-1)^{1/3} \operatorname{Chi}\left(\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right) \cosh\left(c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right)}{9 a^2/3 b^4/3} \\ & + \frac{(-1)^{2/3} \operatorname{Chi}\left(-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right) \cosh\left(c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right)}{9 a^2/3 b^4/3} - \frac{x \cosh(dx + c)}{3 b (b x^3 + a)} \\ & - \frac{(-1)^{2/3} d \cosh\left(c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right) \operatorname{Shi}\left(-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx\right)}{9 a^1/3 b^5/3} - \frac{d \cosh\left(c - \frac{a^{1/3} d}{b^{1/3}}\right) \operatorname{Shi}\left(\frac{a^{1/3} d}{b^{1/3}} + dx\right)}{9 a^1/3 b^5/3} \\ & + \frac{(-1)^{1/3} d \cosh\left(c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right) \operatorname{Shi}\left(\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} + dx\right)}{9 a^1/3 b^5/3} - \frac{d \operatorname{Chi}\left(\frac{a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{a^{1/3} d}{b^{1/3}}\right)}{9 a^1/3 b^5/3} \\ & + \frac{\operatorname{Shi}\left(\frac{a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{a^{1/3} d}{b^{1/3}}\right)}{9 a^2/3 b^4/3} - \frac{(-1)^{2/3} d \operatorname{Chi}\left(\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} - dx\right) \sinh\left(c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right)}{9 a^1/3 b^5/3} \\ & - \frac{(-1)^{1/3} \operatorname{Shi}\left(-\frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c + \frac{(-1)^{1/3} a^{1/3} d}{b^{1/3}}\right)}{9 a^2/3 b^4/3} + \frac{(-1)^{1/3} d \operatorname{Chi}\left(-\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}} - dx\right) \sinh\left(c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right)}{9 a^1/3 b^5/3} \end{aligned}$$

$$+ \frac{(-1)^2 / 3 \operatorname{Shi}\left(\frac{(-1)^2 / 3 a^{1/3} d}{b^{1/3}} + dx\right) \sinh\left(c - \frac{(-1)^2 / 3 a^{1/3} d}{b^{1/3}}\right)}{9 a^2 / 3 b^4 / 3}$$

Result (type 7, 876 leaves):

$$-\frac{d^3 e^{-dx-cx}}{6b(bd^3x^3+ad^3)}$$

$$-\frac{1}{18dab^2} \left($$

$$\sum_{RI=RootOf(_Z^3 b-3_Z^2 bc+3_Zb^2+a d^3-b c^3)} \frac{(3_RI^2 b c^2 -_RI a d^3 -5_RI b c^3 -2 a c d^3 +2 b c^4 +3_RI b c^2 + a d^3 - b c^3) e^{-RI} Ei_1(dx -_RI + c)}{_RI^2 -2_RI c + c^2} \right)$$

$$+ \frac{c^3 \left(\sum_{RI=RootOf(_Z^3 b-3_Z^2 bc+3_Zb^2+a d^3-b c^3)} \frac{(_RI - c + 2) e^{-RI} Ei_1(dx -_RI + c)}{_RI^2 -2_RI c + c^2} \right)}{18dab}$$

$$- \frac{c^2 \left(\sum_{RI=RootOf(_Z^3 b-3_Z^2 bc+3_Zb^2+a d^3-b c^3)} \frac{(_RI^2 -_RI c +_RI + c) e^{-RI} Ei_1(dx -_RI + c)}{_RI^2 -2_RI c + c^2} \right)}{6dab}$$

$$+ \frac{c \left(\sum_{RI=RootOf(_Z^3 b-3_Z^2 bc+3_Zb^2+a d^3-b c^3)} \frac{(2_RI^2 bc -3_RI b c^2 - a d^3 + b c^3 + 2_RI bc) e^{-RI} Ei_1(dx -_RI + c)}{_RI^2 -2_RI c + c^2} \right)}{6dab^2} - \frac{d^3 e^{dx+cx}}{6b(bd^3x^3+ad^3)}$$

$$+ \frac{1}{18dab^2} \left($$

$$\sum_{RI=RootOf(_Z^3 b-3_Z^2 bc+3_Zb^2+a d^3-b c^3)} \frac{(3_RI^2 b c^2 -_RI a d^3 -5_RI b c^3 -2 a c d^3 +2 b c^4 -3_RI b c^2 - a d^3 + b c^3) e^{RI} Ei_1(-dx +_RI - c)}{_RI^2 -2_RI c + c^2} \right)$$

$$- \frac{c^3 \left(\sum_{RI=RootOf(_Z^3 b-3_Z^2 bc+3_Zb^2+a d^3-b c^3)} \frac{(_RI - c - 2) e^{RI} Ei_1(-dx +_RI - c)}{_RI^2 -2_RI c + c^2} \right)}{18dab}$$

$$+ \frac{c^2 \left(\sum_{RI=RootOf(_Z^3 b-3_Z^2 bc+3_Zb^2+a d^3-b c^3)} \frac{(_RI^2 -_RI c -_RI - c) e^{RI} Ei_1(-dx +_RI - c)}{_RI^2 -2_RI c + c^2} \right)}{6dab}$$

$$- \frac{c \left(\sum_{RI=RootOf(_Z^3 b-3_Z^2 bc+3_Zb^2+a d^3-b c^3)} \frac{(2_RI^2 bc -3_RI b c^2 - a d^3 + b c^3 - 2_RI bc) e^{RI} Ei_1(-dx +_RI - c)}{_RI^2 -2_RI c + c^2} \right)}{6dab^2}$$

Test results for the 22 problems in "6.2.3 (e x)^m (a+b cosh(c+d x^n))^p.txt"

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx$$

Optimal(type 3, 46 leaves, 4 steps):

$$\frac{2 \cosh\left(a + \frac{b}{x}\right)}{b^2 x} - \frac{2 \sinh\left(a + \frac{b}{x}\right)}{b^3} - \frac{\sinh\left(a + \frac{b}{x}\right)}{b x^2}$$

Result(type 3, 93 leaves):

$$\frac{\left(a + \frac{b}{x}\right)^2 \sinh\left(a + \frac{b}{x}\right) - 2 \left(a + \frac{b}{x}\right) \cosh\left(a + \frac{b}{x}\right) + 2 \sinh\left(a + \frac{b}{x}\right) - 2a \left(\left(a + \frac{b}{x}\right) \sinh\left(a + \frac{b}{x}\right) - \cosh\left(a + \frac{b}{x}\right) \right) + a^2 \sinh\left(a + \frac{b}{x}\right)}{b^3}$$

Problem 12: Result unnecessarily involves higher level functions.

$$\int \cosh(a + b x^n) dx$$

Optimal(type 4, 61 leaves, 3 steps):

$$-\frac{e^a x \Gamma\left(\frac{1}{n}, -b x^n\right)}{2 n (-b x^n)^{\frac{1}{n}}} - \frac{x \Gamma\left(\frac{1}{n}, b x^n\right)}{2 e^a n (b x^n)^{\frac{1}{n}}}$$

Result(type 5, 73 leaves):

$$x \operatorname{hypergeom}\left(\left[\frac{1}{2n}\right], \left[\frac{1}{2}, 1 + \frac{1}{2n}\right], \frac{x^{2n} b^2}{4}\right) \cosh(a) + \frac{x^{n+1} b \operatorname{hypergeom}\left(\left[\frac{1}{2} + \frac{1}{2n}\right], \left[\frac{3}{2}, \frac{3}{2} + \frac{1}{2n}\right], \frac{x^{2n} b^2}{4}\right) \sinh(a)}{n+1}$$

Problem 13: Unable to integrate problem.

$$\int \cosh(a + b x^n)^3 dx$$

Optimal(type 4, 140 leaves, 8 steps):

$$-\frac{e^{3a} x \Gamma\left(\frac{1}{n}, -3 b x^n\right)}{8 3^n n (-b x^n)^{\frac{1}{n}}} - \frac{3 e^a x \Gamma\left(\frac{1}{n}, -b x^n\right)}{8 n (-b x^n)^{\frac{1}{n}}} - \frac{3 x \Gamma\left(\frac{1}{n}, b x^n\right)}{8 e^a n (b x^n)^{\frac{1}{n}}} - \frac{x \Gamma\left(\frac{1}{n}, 3 b x^n\right)}{8 3^n e^{3a} n (b x^n)^{\frac{1}{n}}}$$

Result(type 8, 12 leaves):

$$\int \cosh(a + b x^n)^3 dx$$

Problem 15: Unable to integrate problem.

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx$$

Optimal(type 6, 117 leaves, 5 steps):

$$\frac{(ex)^n \text{AppellF1}\left(\frac{1}{2}, -p, \frac{1}{2}, \frac{3}{2}, \frac{b(1 - \cosh(c + dx^n))}{a + b}, \frac{1}{2} - \frac{\cosh(c + dx^n)}{2}\right) (a + b \cosh(c + dx^n))^p \sinh(c + dx^n) \sqrt{2}}{den x^n \left(\frac{a + b \cosh(c + dx^n)}{a + b}\right)^p \sqrt{1 + \cosh(c + dx^n)}}$$

Result(type 8, 24 leaves):

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx$$

Problem 16: Result unnecessarily involves higher level functions.

$$\int x^m \cosh(a + bx^n) dx$$

Optimal(type 4, 85 leaves, 3 steps):

$$-\frac{e^a x^{1+m} \Gamma\left(\frac{1+m}{n}, -bx^n\right)}{2n (-bx^n)^{\frac{1+m}{n}}} - \frac{x^{1+m} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{2e^a n (bx^n)^{\frac{1+m}{n}}}$$

Result(type 5, 109 leaves):

$$\frac{x^{1+m} \text{hypergeom}\left(\left[\frac{m}{2n} + \frac{1}{2n}\right], \left[\frac{1}{2}, 1 + \frac{m}{2n} + \frac{1}{2n}\right], \frac{x^{2n} b^2}{4}\right) \cosh(a)}{1+m} + \frac{x^{m+n+1} b \text{hypergeom}\left(\left[\frac{1}{2} + \frac{m}{2n} + \frac{1}{2n}\right], \left[\frac{3}{2}, \frac{3}{2} + \frac{m}{2n} + \frac{1}{2n}\right], \frac{x^{2n} b^2}{4}\right) \sinh(a)}{m+n+1}$$

Problem 17: Unable to integrate problem.

$$\int x^m \cosh(a + bx^n)^3 dx$$

Optimal(type 4, 196 leaves, 8 steps):

$$-\frac{e^3 a x^{1+m} \Gamma\left(\frac{1+m}{n}, -3bx^n\right)}{83 \frac{1+m}{n} n (-bx^n)^{\frac{1+m}{n}}} - \frac{3e^a x^{1+m} \Gamma\left(\frac{1+m}{n}, -bx^n\right)}{8n (-bx^n)^{\frac{1+m}{n}}} - \frac{3x^{1+m} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{8e^a n (bx^n)^{\frac{1+m}{n}}} - \frac{x^{1+m} \Gamma\left(\frac{1+m}{n}, 3bx^n\right)}{83 \frac{1+m}{n} e^3 a n (bx^n)^{\frac{1+m}{n}}}$$

Result(type 8, 16 leaves):

$$\int x^m \cosh(a + bx^n)^3 dx$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int x^2 \cosh(a + b\sqrt{dx+c}) dx$$

Optimal (type 3, 310 leaves, 16 steps):

$$\begin{aligned} & -\frac{240 \cosh(a + b\sqrt{dx+c})}{b^6 d^3} + \frac{24c \cosh(a + b\sqrt{dx+c})}{b^4 d^3} - \frac{2c^2 \cosh(a + b\sqrt{dx+c})}{b^2 d^3} - \frac{120(dx+c) \cosh(a + b\sqrt{dx+c})}{b^4 d^3} \\ & + \frac{12c(dx+c) \cosh(a + b\sqrt{dx+c})}{b^2 d^3} - \frac{10(dx+c)^2 \cosh(a + b\sqrt{dx+c})}{b^2 d^3} + \frac{40(dx+c)^{3/2} \sinh(a + b\sqrt{dx+c})}{b^3 d^3} \\ & - \frac{4c(dx+c)^{3/2} \sinh(a + b\sqrt{dx+c})}{b d^3} + \frac{2(dx+c)^{5/2} \sinh(a + b\sqrt{dx+c})}{b d^3} + \frac{240 \sinh(a + b\sqrt{dx+c}) \sqrt{dx+c}}{b^5 d^3} \\ & - \frac{24c \sinh(a + b\sqrt{dx+c}) \sqrt{dx+c}}{b^3 d^3} + \frac{2c^2 \sinh(a + b\sqrt{dx+c}) \sqrt{dx+c}}{b d^3} \end{aligned}$$

Result (type 3, 830 leaves):

$$\begin{aligned} & \frac{1}{d^3 b^2} \left(2 \left(\frac{1}{b^4} \left((a + b\sqrt{dx+c})^5 \sinh(a + b\sqrt{dx+c}) - 5(a + b\sqrt{dx+c})^4 \cosh(a + b\sqrt{dx+c}) + 20(a + b\sqrt{dx+c})^3 \sinh(a + b\sqrt{dx+c}) \right. \right. \right. \\ & - 60(a + b\sqrt{dx+c})^2 \cosh(a + b\sqrt{dx+c}) + 120(a + b\sqrt{dx+c}) \sinh(a + b\sqrt{dx+c}) - 120 \cosh(a + b\sqrt{dx+c}) \left. \left. \left. \right) - \frac{1}{b^4} \left(5a \left((a \right. \right. \right. \right. \\ & + b\sqrt{dx+c})^4 \sinh(a + b\sqrt{dx+c}) - 4(a + b\sqrt{dx+c})^3 \cosh(a + b\sqrt{dx+c}) + 12(a + b\sqrt{dx+c})^2 \sinh(a + b\sqrt{dx+c}) - 24(a \\ & + b\sqrt{dx+c}) \cosh(a + b\sqrt{dx+c}) + 24 \sinh(a + b\sqrt{dx+c}) \left. \left. \left. \right) \right) + \frac{1}{b^4} \left(10a^2 \left((a + b\sqrt{dx+c})^3 \sinh(a + b\sqrt{dx+c}) - 3(a \right. \right. \right. \\ & + b\sqrt{dx+c})^2 \cosh(a + b\sqrt{dx+c}) + 6(a + b\sqrt{dx+c}) \sinh(a + b\sqrt{dx+c}) - 6 \cosh(a + b\sqrt{dx+c}) \left. \left. \left. \right) \right) \right) \\ & - \frac{10a^3 \left((a + b\sqrt{dx+c})^2 \sinh(a + b\sqrt{dx+c}) - 2(a + b\sqrt{dx+c}) \cosh(a + b\sqrt{dx+c}) + 2 \sinh(a + b\sqrt{dx+c}) \right)}{b^4} - \frac{1}{b^2} \left(2c \left((a \right. \right. \right. \\ & + b\sqrt{dx+c})^3 \sinh(a + b\sqrt{dx+c}) - 3(a + b\sqrt{dx+c})^2 \cosh(a + b\sqrt{dx+c}) + 6(a + b\sqrt{dx+c}) \sinh(a + b\sqrt{dx+c}) - 6 \cosh(a \\ & + b\sqrt{dx+c}) \left. \left. \left. \right) \right) + \frac{6ca \left((a + b\sqrt{dx+c})^2 \sinh(a + b\sqrt{dx+c}) - 2(a + b\sqrt{dx+c}) \cosh(a + b\sqrt{dx+c}) + 2 \sinh(a + b\sqrt{dx+c}) \right)}{b^2} \\ & + \frac{5a^4 \left((a + b\sqrt{dx+c}) \sinh(a + b\sqrt{dx+c}) - \cosh(a + b\sqrt{dx+c}) \right)}{b^4} \\ & - \frac{6a^2 c \left((a + b\sqrt{dx+c}) \sinh(a + b\sqrt{dx+c}) - \cosh(a + b\sqrt{dx+c}) \right)}{b^2} - \frac{a^5 \sinh(a + b\sqrt{dx+c})}{b^4} + \frac{2a^3 c \sinh(a + b\sqrt{dx+c})}{b^2} + c^2 \left((a \right. \end{aligned}$$

$$+ b \sqrt{dx+c} \sinh(a+b\sqrt{dx+c}) - \cosh(a+b\sqrt{dx+c}) - c^2 a \sinh(a+b\sqrt{dx+c}) \Big) \Big)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int x^2 \cosh(a+b(dx+c)^{1/3}) dx$$

Optimal (type 3, 477 leaves, 23 steps):

$$\begin{aligned} & \frac{720 c \cosh(a+b(dx+c)^{1/3})}{b^6 d^3} - \frac{120960 (dx+c)^{1/3} \cosh(a+b(dx+c)^{1/3})}{b^8 d^3} - \frac{6 c^2 (dx+c)^{1/3} \cosh(a+b(dx+c)^{1/3})}{b^2 d^3} \\ & + \frac{360 c (dx+c)^{2/3} \cosh(a+b(dx+c)^{1/3})}{b^4 d^3} - \frac{20160 (dx+c) \cosh(a+b(dx+c)^{1/3})}{b^6 d^3} + \frac{30 c (dx+c)^{4/3} \cosh(a+b(dx+c)^{1/3})}{b^2 d^3} \\ & - \frac{1008 (dx+c)^{5/3} \cosh(a+b(dx+c)^{1/3})}{b^4 d^3} - \frac{24 (dx+c)^{7/3} \cosh(a+b(dx+c)^{1/3})}{b^2 d^3} + \frac{120960 \sinh(a+b(dx+c)^{1/3})}{b^9 d^3} \\ & + \frac{6 c^2 \sinh(a+b(dx+c)^{1/3})}{b^3 d^3} - \frac{720 c (dx+c)^{1/3} \sinh(a+b(dx+c)^{1/3})}{b^5 d^3} + \frac{60480 (dx+c)^{2/3} \sinh(a+b(dx+c)^{1/3})}{b^7 d^3} \\ & + \frac{3 c^2 (dx+c)^{2/3} \sinh(a+b(dx+c)^{1/3})}{b d^3} - \frac{120 c (dx+c) \sinh(a+b(dx+c)^{1/3})}{b^3 d^3} + \frac{5040 (dx+c)^{4/3} \sinh(a+b(dx+c)^{1/3})}{b^5 d^3} \\ & - \frac{6 c (dx+c)^{5/3} \sinh(a+b(dx+c)^{1/3})}{b d^3} + \frac{168 (dx+c)^2 \sinh(a+b(dx+c)^{1/3})}{b^3 d^3} + \frac{3 (dx+c)^{8/3} \sinh(a+b(dx+c)^{1/3})}{b d^3} \end{aligned}$$

Result (type 3, 1814 leaves):

$$\begin{aligned} & \frac{1}{d^3 b^3} \left(3 \left(a^2 c^2 \sinh(a+b(dx+c)^{1/3}) - \frac{1}{b^6} (8 a ((a+b(dx+c)^{1/3})^7 \sinh(a+b(dx+c)^{1/3}) - 7 (a+b(dx+c)^{1/3})^6 \cosh(a+b(dx+c)^{1/3}) \right. \right. \\ & + 42 (a+b(dx+c)^{1/3})^5 \sinh(a+b(dx+c)^{1/3}) - 210 (a+b(dx+c)^{1/3})^4 \cosh(a+b(dx+c)^{1/3}) + 840 (a+b(dx+c)^{1/3})^3 \sinh(a \\ & + b(dx+c)^{1/3}) - 2520 (a+b(dx+c)^{1/3})^2 \cosh(a+b(dx+c)^{1/3}) + 5040 (a+b(dx+c)^{1/3}) \sinh(a+b(dx+c)^{1/3}) - 5040 \cosh(a \\ & + b(dx+c)^{1/3}) \Big) \Big) + \frac{1}{b^6} (28 a^2 ((a+b(dx+c)^{1/3})^6 \sinh(a+b(dx+c)^{1/3}) - 6 (a+b(dx+c)^{1/3})^5 \cosh(a+b(dx+c)^{1/3}) \\ & + 30 (a+b(dx+c)^{1/3})^4 \sinh(a+b(dx+c)^{1/3}) - 120 (a+b(dx+c)^{1/3})^3 \cosh(a+b(dx+c)^{1/3}) + 360 (a+b(dx+c)^{1/3})^2 \sinh(a \\ & + b(dx+c)^{1/3}) - 720 (a+b(dx+c)^{1/3}) \cosh(a+b(dx+c)^{1/3}) + 720 \sinh(a+b(dx+c)^{1/3})) \Big) - \frac{1}{b^6} (56 a^3 ((a+b(dx \\ & + c)^{1/3})^5 \sinh(a+b(dx+c)^{1/3}) - 5 (a+b(dx+c)^{1/3})^4 \cosh(a+b(dx+c)^{1/3}) + 20 (a+b(dx+c)^{1/3})^3 \sinh(a+b(dx+c)^{1/3}) \end{aligned}$$

$$\begin{aligned}
& -60 (a + b (dx + c)^{1/3})^2 \cosh(a + b (dx + c)^{1/3}) + 120 (a + b (dx + c)^{1/3}) \sinh(a + b (dx + c)^{1/3}) - 120 \cosh(a + b (dx + c)^{1/3}) \Big) \\
& + \frac{1}{b^6} (70 a^4 (a + b (dx + c)^{1/3})^4 \sinh(a + b (dx + c)^{1/3}) - 4 (a + b (dx + c)^{1/3})^3 \cosh(a + b (dx + c)^{1/3}) + 12 (a + b (dx + c)^{1/3})^2 \sinh(a \\
& + b (dx + c)^{1/3}) - 24 (a + b (dx + c)^{1/3}) \cosh(a + b (dx + c)^{1/3}) + 24 \sinh(a + b (dx + c)^{1/3}) \Big) - 2 a c^2 (a + b (dx + c)^{1/3}) \sinh(a \\
& + b (dx + c)^{1/3}) - \cosh(a + b (dx + c)^{1/3}) \Big) - \frac{1}{b^6} (56 a^5 (a + b (dx + c)^{1/3})^3 \sinh(a + b (dx + c)^{1/3}) - 3 (a + b (dx + c)^{1/3})^2 \cosh(a \\
& + b (dx + c)^{1/3}) + 6 (a + b (dx + c)^{1/3}) \sinh(a + b (dx + c)^{1/3}) - 6 \cosh(a + b (dx + c)^{1/3}) \Big) - \frac{1}{b^3} (2 c (a + b (dx + c)^{1/3})^5 \sinh(a \\
& + b (dx + c)^{1/3}) - 5 (a + b (dx + c)^{1/3})^4 \cosh(a + b (dx + c)^{1/3}) + 20 (a + b (dx + c)^{1/3})^3 \sinh(a + b (dx + c)^{1/3}) - 60 (a + b (dx \\
& + c)^{1/3})^2 \cosh(a + b (dx + c)^{1/3}) + 120 (a + b (dx + c)^{1/3}) \sinh(a + b (dx + c)^{1/3}) - 120 \cosh(a + b (dx + c)^{1/3}) \Big) \\
& + \frac{28 a^6 (a + b (dx + c)^{1/3})^2 \sinh(a + b (dx + c)^{1/3}) - 2 (a + b (dx + c)^{1/3}) \cosh(a + b (dx + c)^{1/3}) + 2 \sinh(a + b (dx + c)^{1/3}) \Big)}{b^6} \\
& - \frac{8 a^7 (a + b (dx + c)^{1/3}) \sinh(a + b (dx + c)^{1/3}) - \cosh(a + b (dx + c)^{1/3}) \Big)}{b^6} + \frac{2 a^5 c \sinh(a + b (dx + c)^{1/3})}{b^3} + c^2 (a + b (dx \\
& + c)^{1/3})^2 \sinh(a + b (dx + c)^{1/3}) - 2 (a + b (dx + c)^{1/3}) \cosh(a + b (dx + c)^{1/3}) + 2 \sinh(a + b (dx + c)^{1/3}) \Big) + \frac{1}{b^6} (a \\
& + b (dx + c)^{1/3})^8 \sinh(a + b (dx + c)^{1/3}) - 8 (a + b (dx + c)^{1/3})^7 \cosh(a + b (dx + c)^{1/3}) + 56 (a + b (dx + c)^{1/3})^6 \sinh(a + b (dx + c)^{1/3}) \\
& - 336 (a + b (dx + c)^{1/3})^5 \cosh(a + b (dx + c)^{1/3}) + 1680 (a + b (dx + c)^{1/3})^4 \sinh(a + b (dx + c)^{1/3}) - 6720 (a + b (dx + c)^{1/3})^3 \cosh(a \\
& + b (dx + c)^{1/3}) + 20160 (a + b (dx + c)^{1/3})^2 \sinh(a + b (dx + c)^{1/3}) - 40320 (a + b (dx + c)^{1/3}) \cosh(a + b (dx + c)^{1/3}) + 40320 \sinh(a \\
& + b (dx + c)^{1/3}) \Big) + \frac{a^8 \sinh(a + b (dx + c)^{1/3})}{b^6} + \frac{1}{b^3} (10 c a (a + b (dx + c)^{1/3})^4 \sinh(a + b (dx + c)^{1/3}) - 4 (a + b (dx \\
& + c)^{1/3})^3 \cosh(a + b (dx + c)^{1/3}) + 12 (a + b (dx + c)^{1/3})^2 \sinh(a + b (dx + c)^{1/3}) - 24 (a + b (dx + c)^{1/3}) \cosh(a + b (dx + c)^{1/3}) \\
& + 24 \sinh(a + b (dx + c)^{1/3}) \Big) - \frac{1}{b^3} (20 c a^2 (a + b (dx + c)^{1/3})^3 \sinh(a + b (dx + c)^{1/3}) - 3 (a + b (dx + c)^{1/3})^2 \cosh(a + b (dx
\end{aligned}$$

$$\begin{aligned}
& +c)^{1/3}) + 6(a + b(dx + c)^{1/3}) \sinh(a + b(dx + c)^{1/3}) - 6 \cosh(a + b(dx + c)^{1/3})) \\
& + \frac{20a^3c((a + b(dx + c)^{1/3})^2 \sinh(a + b(dx + c)^{1/3}) - 2(a + b(dx + c)^{1/3}) \cosh(a + b(dx + c)^{1/3}) + 2 \sinh(a + b(dx + c)^{1/3}))}{b^3} \\
& - \frac{10a^4c((a + b(dx + c)^{1/3}) \sinh(a + b(dx + c)^{1/3}) - \cosh(a + b(dx + c)^{1/3}))}{b^3} \Big) \Big)
\end{aligned}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int x \cosh(a + b(dx + c)^{1/3}) dx$$

Optimal (type 3, 231 leaves, 13 steps):

$$\begin{aligned}
& - \frac{360 \cosh(a + b(dx + c)^{1/3})}{b^6 d^2} + \frac{6c(dx + c)^{1/3} \cosh(a + b(dx + c)^{1/3})}{b^2 d^2} - \frac{180(dx + c)^{2/3} \cosh(a + b(dx + c)^{1/3})}{b^4 d^2} \\
& - \frac{15(dx + c)^{4/3} \cosh(a + b(dx + c)^{1/3})}{b^2 d^2} - \frac{6c \sinh(a + b(dx + c)^{1/3})}{b^3 d^2} + \frac{360(dx + c)^{1/3} \sinh(a + b(dx + c)^{1/3})}{b^5 d^2} \\
& - \frac{3c(dx + c)^{2/3} \sinh(a + b(dx + c)^{1/3})}{b d^2} + \frac{60(dx + c) \sinh(a + b(dx + c)^{1/3})}{b^3 d^2} + \frac{3(dx + c)^{5/3} \sinh(a + b(dx + c)^{1/3})}{b d^2}
\end{aligned}$$

Result (type 3, 658 leaves):

$$\begin{aligned}
& \frac{1}{d^2 b^3} \left(3 \left(\frac{1}{b^3} \left((a + b(dx + c)^{1/3})^5 \sinh(a + b(dx + c)^{1/3}) - 5(a + b(dx + c)^{1/3})^4 \cosh(a + b(dx + c)^{1/3}) + 20(a + b(dx + c)^{1/3})^3 \sinh(a \right. \right. \right. \\
& + b(dx + c)^{1/3}) - 60(a + b(dx + c)^{1/3})^2 \cosh(a + b(dx + c)^{1/3}) + 120(a + b(dx + c)^{1/3}) \sinh(a + b(dx + c)^{1/3}) - 120 \cosh(a \\
& + b(dx + c)^{1/3}) \Big) - \frac{1}{b^3} \left(5a \left((a + b(dx + c)^{1/3})^4 \sinh(a + b(dx + c)^{1/3}) - 4(a + b(dx + c)^{1/3})^3 \cosh(a + b(dx + c)^{1/3}) \right. \right. \\
& + 12(a + b(dx + c)^{1/3})^2 \sinh(a + b(dx + c)^{1/3}) - 24(a + b(dx + c)^{1/3}) \cosh(a + b(dx + c)^{1/3}) + 24 \sinh(a + b(dx + c)^{1/3}) \Big) \Big) \\
& + \frac{1}{b^3} \left(10a^2 \left((a + b(dx + c)^{1/3})^3 \sinh(a + b(dx + c)^{1/3}) - 3(a + b(dx + c)^{1/3})^2 \cosh(a + b(dx + c)^{1/3}) + 6(a + b(dx + c)^{1/3}) \sinh(a \right. \right. \\
& + b(dx + c)^{1/3}) - 6 \cosh(a + b(dx + c)^{1/3}) \Big) \Big) \\
& - \frac{10a^3 \left((a + b(dx + c)^{1/3})^2 \sinh(a + b(dx + c)^{1/3}) - 2(a + b(dx + c)^{1/3}) \cosh(a + b(dx + c)^{1/3}) + 2 \sinh(a + b(dx + c)^{1/3}) \right)}{b^3} \\
& + \frac{5a^4 \left((a + b(dx + c)^{1/3}) \sinh(a + b(dx + c)^{1/3}) - \cosh(a + b(dx + c)^{1/3}) \right)}{b^3} - \frac{a^5 \sinh(a + b(dx + c)^{1/3})}{b^3} - c \left((a + b(dx
\end{aligned}$$

$$+c)^{1/3})^2 \sinh(a+b(dx+c)^{1/3}) - 2(a+b(dx+c)^{1/3}) \cosh(a+b(dx+c)^{1/3}) + 2 \sinh(a+b(dx+c)^{1/3}) + 2ac((a+b(dx+c)^{1/3}) \sinh(a+b(dx+c)^{1/3}) - \cosh(a+b(dx+c)^{1/3})) - a^2c \sinh(a+b(dx+c)^{1/3}))$$

Test results for the 11 problems in "6.2.4 (d+e x)^m cosh(a+b x+c x^2)^n.txt"

Problem 4: Unable to integrate problem.

$$\int \left(\frac{\cosh(-cx^2+bx+a)}{x^2} - \frac{b \sinh(-cx^2+bx+a)}{x} \right) dx$$

Optimal(type 4, 82 leaves, 7 steps):

$$-\frac{\cosh(-cx^2+bx+a)}{x} + \frac{e^{a+\frac{b^2}{4c}} \operatorname{erf}\left(\frac{-2cx+b}{2\sqrt{c}}\right) \sqrt{c} \sqrt{\pi}}{2} - \frac{e^{-a-\frac{b^2}{4c}} \operatorname{erfi}\left(\frac{-2cx+b}{2\sqrt{c}}\right) \sqrt{c} \sqrt{\pi}}{2}$$

Result(type 8, 37 leaves):

$$\int \left(\frac{\cosh(-cx^2+bx+a)}{x^2} - \frac{b \sinh(-cx^2+bx+a)}{x} \right) dx$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int (ex+d) \cosh(cx^2+bx+a) dx$$

Optimal(type 4, 100 leaves, 6 steps):

$$\frac{e \sinh(cx^2+bx+a)}{2c} + \frac{(-be+2cd) e^{-a+\frac{b^2}{4c}} \operatorname{erf}\left(\frac{2cx+b}{2\sqrt{c}}\right) \sqrt{\pi}}{8c^3/2} + \frac{(-be+2cd) e^{a-\frac{b^2}{4c}} \operatorname{erfi}\left(\frac{2cx+b}{2\sqrt{c}}\right) \sqrt{\pi}}{8c^3/2}$$

Result(type 4, 210 leaves):

$$\frac{d\sqrt{\pi} e^{-\frac{4ca-b^2}{4c}} \operatorname{erf}\left(\sqrt{c}x + \frac{b}{2\sqrt{c}}\right)}{4\sqrt{c}} - \frac{e e^{-cx^2-bx-a}}{4c} - \frac{eb\sqrt{\pi} e^{-\frac{4ca-b^2}{4c}} \operatorname{erf}\left(\sqrt{c}x + \frac{b}{2\sqrt{c}}\right)}{8c^3/2} - \frac{d\sqrt{\pi} e^{\frac{4ca-b^2}{4c}} \operatorname{erf}\left(-\sqrt{-c}x + \frac{b}{2\sqrt{-c}}\right)}{4\sqrt{-c}} + \frac{e e^{cx^2+bx+a}}{4c} + \frac{eb\sqrt{\pi} e^{\frac{4ca-b^2}{4c}} \operatorname{erf}\left(-\sqrt{-c}x + \frac{b}{2\sqrt{-c}}\right)}{8c\sqrt{-c}}$$

Test results for the 89 problems in "6.2.5 Hyperbolic cosine functions.txt"

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \cosh(bx + a)^{7/2} dx$$

Optimal(type 4, 85 leaves, 3 steps):

$$-\frac{10 \operatorname{I} \sqrt{\cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^2} \operatorname{EllipticF}\left(\operatorname{I} \sinh\left(\frac{bx}{2} + \frac{a}{2}\right), \sqrt{2}\right)}{21 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right) b} + \frac{2 \cosh(bx + a)^{5/2} \sinh(bx + a)}{7b} + \frac{10 \sinh(bx + a) \sqrt{\cosh(bx + a)}}{21b}$$

Result(type 4, 200 leaves):

$$\left(2 \sqrt{\left(2 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^2 - 1\right) \sinh\left(\frac{bx}{2} + \frac{a}{2}\right)^2} \left(48 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^9 - 120 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^7 + 128 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^5 - 72 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^3 \right. \right. \\ \left. \left. + 5 \sqrt{-\sinh\left(\frac{bx}{2} + \frac{a}{2}\right)^2} \sqrt{-2 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^2 + 1} \operatorname{EllipticF}\left(\cosh\left(\frac{bx}{2} + \frac{a}{2}\right), \sqrt{2}\right) + 16 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right) \right) \right) / \\ \left(21 \sqrt{2 \sinh\left(\frac{bx}{2} + \frac{a}{2}\right)^4 + \sinh\left(\frac{bx}{2} + \frac{a}{2}\right)^2} \sinh\left(\frac{bx}{2} + \frac{a}{2}\right) \sqrt{2 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^2 - 1} b \right)$$

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\cosh(bx + a)} dx$$

Optimal(type 4, 46 leaves, 1 step):

$$\frac{-2 \operatorname{I} \sqrt{\cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^2} \operatorname{EllipticE}\left(\operatorname{I} \sinh\left(\frac{bx}{2} + \frac{a}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{bx}{2} + \frac{a}{2}\right) b}$$

Result(type 4, 134 leaves):

$$-\frac{2 \sqrt{\left(2 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^2 - 1\right) \sinh\left(\frac{bx}{2} + \frac{a}{2}\right)^2} \sqrt{-\sinh\left(\frac{bx}{2} + \frac{a}{2}\right)^2} \sqrt{-2 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^2 + 1} \operatorname{EllipticE}\left(\cosh\left(\frac{bx}{2} + \frac{a}{2}\right), \sqrt{2}\right)}{\sqrt{2 \sinh\left(\frac{bx}{2} + \frac{a}{2}\right)^4 + \sinh\left(\frac{bx}{2} + \frac{a}{2}\right)^2} \sinh\left(\frac{bx}{2} + \frac{a}{2}\right) \sqrt{2 \cosh\left(\frac{bx}{2} + \frac{a}{2}\right)^2 - 1} b}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int (a \cosh(x))^{7/2} dx$$

Optimal(type 4, 66 leaves, 4 steps):

$$\frac{2 a (a \cosh(x))^5 / 2 \sinh(x)}{7} - \frac{10 I a^4 \sqrt{\cosh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{\cosh(x)}}{21 \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x)}} + \frac{10 a^3 \sinh(x) \sqrt{a \cosh(x)}}{21}$$

Result(type 4, 144 leaves):

$$\frac{1}{21 \sqrt{a \left(2 \sinh\left(\frac{x}{2}\right)^4 + \sinh\left(\frac{x}{2}\right)^2\right)} \sinh\left(\frac{x}{2}\right) \sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1\right)}} \left(\sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1\right)} \sinh\left(\frac{x}{2}\right)^2 a^4 \left(96 \cosh\left(\frac{x}{2}\right)^9 - 240 \cosh\left(\frac{x}{2}\right)^7\right) \right. \\ \left. + 256 \cosh\left(\frac{x}{2}\right)^5 + 5 \sqrt{2} \sqrt{-2 \cosh\left(\frac{x}{2}\right)^2 + 1} \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{2}, \frac{\sqrt{2}}{2}\right) - 144 \cosh\left(\frac{x}{2}\right)^3 + 32 \cosh\left(\frac{x}{2}\right) \right)$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int (a \cosh(x))^5 / 2 \, dx$$

Optimal(type 4, 53 leaves, 3 steps):

$$\frac{2 a (a \cosh(x))^3 / 2 \sinh(x)}{5} - \frac{6 I a^2 \sqrt{\cosh\left(\frac{x}{2}\right)^2} \operatorname{EllipticE}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x)}}{5 \cosh\left(\frac{x}{2}\right) \sqrt{\cosh(x)}}$$

Result(type 4, 183 leaves):

$$\frac{1}{5 \sqrt{a \left(2 \sinh\left(\frac{x}{2}\right)^4 + \sinh\left(\frac{x}{2}\right)^2\right)} \sinh\left(\frac{x}{2}\right) \sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1\right)}} \left(\sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1\right)} \sinh\left(\frac{x}{2}\right)^2 a^3 \left(16 \cosh\left(\frac{x}{2}\right) \sinh\left(\frac{x}{2}\right)^6\right) \right. \\ \left. + 16 \sinh\left(\frac{x}{2}\right)^4 \cosh\left(\frac{x}{2}\right) + 3 \sqrt{2} \sqrt{-2 \sinh\left(\frac{x}{2}\right)^2 - 1} \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{2}, \frac{\sqrt{2}}{2}\right) \right. \\ \left. - 6 \sqrt{2} \sqrt{-2 \sinh\left(\frac{x}{2}\right)^2 - 1} \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \operatorname{EllipticE}\left(\cosh\left(\frac{x}{2}\right) \sqrt{2}, \frac{\sqrt{2}}{2}\right) + 4 \sinh\left(\frac{x}{2}\right)^2 \cosh\left(\frac{x}{2}\right) \right)$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int (a \cosh(x))^3 / 2 \, dx$$

Optimal(type 4, 53 leaves, 3 steps):

$$-\frac{2 I a^2 \sqrt{\cosh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{\cosh(x)}}{3 \cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x)}} + \frac{2 a \sinh(x) \sqrt{a \cosh(x)}}{3}$$

Result(type 4, 129 leaves):

$$\frac{1}{3 \sqrt{a \left(2 \sinh\left(\frac{x}{2}\right)^4 + \sinh\left(\frac{x}{2}\right)^2\right)} \sinh\left(\frac{x}{2}\right) \sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1\right)}} \left(\sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1\right)} \sinh\left(\frac{x}{2}\right)^2 a^2 \left(8 \sinh\left(\frac{x}{2}\right)^4 \cosh\left(\frac{x}{2}\right)\right) + \sqrt{2} \sqrt{-2 \sinh\left(\frac{x}{2}\right)^2 - 1} \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{2}, \frac{\sqrt{2}}{2}\right) + 4 \sinh\left(\frac{x}{2}\right)^2 \cosh\left(\frac{x}{2}\right) \right)$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a \cosh(x)} dx$$

Optimal(type 4, 38 leaves, 2 steps):

$$\frac{-2 I \sqrt{\cosh\left(\frac{x}{2}\right)^2} \operatorname{EllipticE}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x)}}{\cosh\left(\frac{x}{2}\right) \sqrt{\cosh(x)}}$$

Result(type 4, 117 leaves):

$$\frac{\sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1\right)} \sinh\left(\frac{x}{2}\right)^2 a \sqrt{2} \sqrt{-2 \cosh\left(\frac{x}{2}\right)^2 + 1} \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \left(\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{2}, \frac{\sqrt{2}}{2}\right) - 2 \operatorname{EllipticE}\left(\cosh\left(\frac{x}{2}\right) \sqrt{2}, \frac{\sqrt{2}}{2}\right) \right)}{\sqrt{a \left(2 \sinh\left(\frac{x}{2}\right)^4 + \sinh\left(\frac{x}{2}\right)^2\right)} \sinh\left(\frac{x}{2}\right) \sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1\right)}}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a \cosh(x)}} dx$$

Optimal(type 4, 38 leaves, 2 steps):

$$\frac{-2 I \sqrt{\cosh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2}\right) \sqrt{\cosh(x)}}{\cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x)}}$$

Result(type 4, 99 leaves):

$$\frac{\sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1 \right) \sinh\left(\frac{x}{2}\right)^2} \sqrt{2} \sqrt{-2 \cosh\left(\frac{x}{2}\right)^2 + 1} \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{2}, \frac{\sqrt{2}}{2}\right)}{\sqrt{a \left(2 \sinh\left(\frac{x}{2}\right)^4 + \sinh\left(\frac{x}{2}\right)^2 \right) \sinh\left(\frac{x}{2}\right)} \sqrt{a \left(2 \cosh\left(\frac{x}{2}\right)^2 - 1 \right)}$$

Problem 10: Unable to integrate problem.

$$\int (b \cosh(dx + c))^n dx$$

Optimal(type 5, 65 leaves, 1 step):

$$\frac{(b \cosh(dx + c))^{n+1} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{n}{2} + \frac{1}{2}\right], \left[\frac{3}{2} + \frac{n}{2}\right], \cosh(dx + c)^2\right) \sinh(dx + c)}{bd(n+1) \sqrt{-\sinh(dx + c)^2}}$$

Result(type 8, 12 leaves):

$$\int (b \cosh(dx + c))^n dx$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + a \cosh(dx + c)}} dx$$

Optimal(type 3, 37 leaves, 2 steps):

$$\frac{\arctan\left(\frac{\sinh(dx + c) \sqrt{a} \sqrt{2}}{2 \sqrt{a + a \cosh(dx + c)}}\right) \sqrt{2}}{d \sqrt{a}}$$

Result(type 3, 102 leaves):

$$\frac{\cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \sinh\left(\frac{c}{2} + \frac{dx}{2}\right)^2} \ln\left(\frac{2 \left(\sqrt{-a} \sqrt{a \sinh\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - a}\right)}{\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)}\right) \sqrt{2}}{\sqrt{-a} \sinh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{a \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)^2} d}$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \cosh(dx + c))^5 / 2} dx$$

Optimal(type 3, 88 leaves, 4 steps):

$$\frac{\sinh(dx+c)}{4d(a+a\cosh(dx+c))^{5/2}} + \frac{3\sinh(dx+c)}{16ad(a+a\cosh(dx+c))^{3/2}} + \frac{3\arctan\left(\frac{\sinh(dx+c)\sqrt{a}\sqrt{2}}{2\sqrt{a+a\cosh(dx+c)}}\right)\sqrt{2}}{32a^{5/2}d}$$

Result(type 3, 177 leaves):

$$-\frac{1}{32a^3\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sqrt{-a}\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)^2}d\left(\sqrt{a\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\left(3\ln\left(\frac{2\left(\sqrt{-a}\sqrt{a\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a}\right)}{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)a\cosh\left(c\right)\right. \right. \\ \left. \left. \left|2+\frac{dx}{2}\right|^4-3\sqrt{a\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sqrt{-a}-2\sqrt{-a}\sqrt{a\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)\sqrt{2}\right)$$

Problem 24: Result more than twice size of optimal antiderivative.

$$\int(a+b\cosh(x))^3/2\,dx$$

Optimal(type 4, 144 leaves, 6 steps):

$$\frac{2b\sinh(x)\sqrt{a+b\cosh(x)}}{3} - \frac{8Ia\sqrt{\cosh\left(\frac{x}{2}\right)^2}\operatorname{EllipticE}\left(\operatorname{I}\sinh\left(\frac{x}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{a+b\cosh(x)}}{3\cosh\left(\frac{x}{2}\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$+ \frac{2I(a^2-b^2)\sqrt{\cosh\left(\frac{x}{2}\right)^2}\operatorname{EllipticF}\left(\operatorname{I}\sinh\left(\frac{x}{2}\right),\sqrt{2}\sqrt{\frac{b}{a+b}}\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}}{3\cosh\left(\frac{x}{2}\right)\sqrt{a+b\cosh(x)}}$$

Result(type 4, 457 leaves):

$$\left(2\left(4\sqrt{-\frac{2b}{a-b}}b^2\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)^4+\left(2\sqrt{-\frac{2b}{a-b}}ab+2\sqrt{-\frac{2b}{a-b}}b^2\right)\sinh\left(\frac{x}{2}\right)^2\cosh\left(\frac{x}{2}\right)\right.\right. \\ \left. +3a^2\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^2}{a-b}+\frac{a+b}{a-b}\sqrt{-\sinh\left(\frac{x}{2}\right)^2}}\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\sqrt{\frac{-2(a-b)}{b}}\right)\right) \\ \left. +4ab\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^2}{a-b}+\frac{a+b}{a-b}\sqrt{-\sinh\left(\frac{x}{2}\right)^2}}\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\sqrt{\frac{-2(a-b)}{b}}\right)\right)$$

$$\begin{aligned}
& + b^2 \sqrt{\frac{2b \sinh\left(\frac{x}{2}\right)^2}{a-b} + \frac{a+b}{a-b} \sqrt{-\sinh\left(\frac{x}{2}\right)^2}} \operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2(a-b)}{b}}\right) \\
& - 8 \sqrt{\frac{2b \sinh\left(\frac{x}{2}\right)^2}{a-b} + \frac{a+b}{a-b} \sqrt{-\sinh\left(\frac{x}{2}\right)^2}} \operatorname{EllipticE}\left(\cosh\left(\frac{x}{2}\right) \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2(a-b)}{b}}\right) a b \sqrt{\left(2 \cosh\left(\frac{x}{2}\right)^2 b + a - b\right) \sinh\left(\frac{x}{2}\right)^2} \\
& / \left(3 \sqrt{-\frac{2b}{a-b}} \sqrt{2b \sinh\left(\frac{x}{2}\right)^4 + (a+b) \sinh\left(\frac{x}{2}\right)^2} \sinh\left(\frac{x}{2}\right) \sqrt{2 \sinh\left(\frac{x}{2}\right)^2 b + a + b} \right)
\end{aligned}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \cosh(dx + c)} \, dx$$

Optimal (type 4, 86 leaves, 2 steps):

$$\frac{-2I \sqrt{\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)^2} \operatorname{EllipticE}\left(I \sinh\left(\frac{c}{2} + \frac{dx}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(dx + c)}}{\cosh\left(\frac{c}{2} + \frac{dx}{2}\right) d \sqrt{\frac{a + b \cosh(dx + c)}{a + b}}}$$

Result (type 4, 275 leaves):

$$\begin{aligned}
& \left(2 \left(a \operatorname{EllipticF}\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2(a-b)}{b}}\right) + b \operatorname{EllipticF}\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2(a-b)}{b}}\right) - 2b \operatorname{EllipticE}\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2(a-b)}{b}}\right) \right) \right. \\
& \left. + \frac{dx}{2} \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2(a-b)}{b}} \right) \\
& \sqrt{-\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)^2} \sqrt{\frac{2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)^2 b + a - b}{a - b}} \sqrt{\left(2 \cosh\left(\frac{c}{2} + \frac{dx}{2}\right)^2 b + a - b\right) \sinh\left(\frac{c}{2} + \frac{dx}{2}\right)^2} / \\
& \left(\sqrt{-\frac{2b}{a-b}} \sqrt{2b \sinh\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + (a+b) \sinh\left(\frac{c}{2} + \frac{dx}{2}\right)^2} \sinh\left(\frac{c}{2} + \frac{dx}{2}\right) \sqrt{2 \sinh\left(\frac{c}{2} + \frac{dx}{2}\right)^2 b + a + b} \right)
\end{aligned}$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \cosh(x)}{\sqrt{a + a \cosh(x)}} \, dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$\frac{(A - B) \arctan\left(\frac{\sinh(x) \sqrt{a} \sqrt{2}}{2\sqrt{a + a \cosh(x)}}\right) \sqrt{2}}{\sqrt{a}} + \frac{2 B \sinh(x)}{\sqrt{a + a \cosh(x)}}$$

Result(type 3, 127 leaves):

$$\frac{\cosh\left(\frac{x}{2}\right) \sqrt{a \sinh\left(\frac{x}{2}\right)^2} \left(\ln\left(\frac{2\left(\sqrt{-a} \sqrt{a \sinh\left(\frac{x}{2}\right)^2 - a}\right)}{\cosh\left(\frac{x}{2}\right)}\right) A a - 2 B \sqrt{a \sinh\left(\frac{x}{2}\right)^2} \sqrt{-a} - \ln\left(\frac{2\left(\sqrt{-a} \sqrt{a \sinh\left(\frac{x}{2}\right)^2 - a}\right)}{\cosh\left(\frac{x}{2}\right)}\right) B a \right) \sqrt{2}}{\sqrt{-a} a \sinh\left(\frac{x}{2}\right) \sqrt{a \cosh\left(\frac{x}{2}\right)^2}}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \cosh(x)}{(a + a \cosh(x))^3 / 2} dx$$

Optimal(type 3, 50 leaves, 3 steps):

$$\frac{(A - B) \sinh(x)}{2(a + a \cosh(x))^3 / 2} + \frac{(A + 3B) \arctan\left(\frac{\sinh(x) \sqrt{a} \sqrt{2}}{2\sqrt{a + a \cosh(x)}}\right) \sqrt{2}}{4a^3 / 2}$$

Result(type 3, 158 leaves):

$$\frac{1}{4 \cosh\left(\frac{x}{2}\right) a^2 \sqrt{-a} \sinh\left(\frac{x}{2}\right) \sqrt{a \cosh\left(\frac{x}{2}\right)^2}} \left(\sqrt{a \sinh\left(\frac{x}{2}\right)^2} \left(\ln\left(\frac{2\left(\sqrt{-a} \sqrt{a \sinh\left(\frac{x}{2}\right)^2 - a}\right)}{\cosh\left(\frac{x}{2}\right)}\right) A a \cosh\left(\frac{x}{2}\right)^2 \right. \right. \\ \left. \left. + 3 \ln\left(\frac{2\left(\sqrt{-a} \sqrt{a \sinh\left(\frac{x}{2}\right)^2 - a}\right)}{\cosh\left(\frac{x}{2}\right)}\right) B a \cosh\left(\frac{x}{2}\right)^2 - A \sqrt{a \sinh\left(\frac{x}{2}\right)^2} \sqrt{-a} + B \sqrt{a \sinh\left(\frac{x}{2}\right)^2} \sqrt{-a} \right) \sqrt{2} \right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \cosh(x)}{(a + b \cosh(x))^3 / 2} dx$$

Optimal(type 4, 178 leaves, 6 steps):

$$\begin{aligned}
& - \frac{2(Ab - aB) \sinh(x)}{(a^2 - b^2) \sqrt{a + b \cosh(x)}} - \frac{2I(Ab - aB) \sqrt{\cosh\left(\frac{x}{2}\right)^2} \operatorname{EllipticE}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{\cosh\left(\frac{x}{2}\right) b (a^2 - b^2) \sqrt{\frac{a + b \cosh(x)}{a+b}}} \\
& - \frac{2IB \sqrt{\cosh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{\cosh\left(\frac{x}{2}\right) b \sqrt{a + b \cosh(x)}}
\end{aligned}$$

Result (type 4, 482 leaves):

$$\begin{aligned}
& \frac{1}{\sinh\left(\frac{x}{2}\right) \sqrt{2 \sinh\left(\frac{x}{2}\right)^2 b + a + b}} \left(\sqrt{\left(2 \cosh\left(\frac{x}{2}\right)^2 b + a - b\right) \sinh\left(\frac{x}{2}\right)^2} \left(\frac{1}{b \sqrt{-\frac{2b}{a-b}} \sqrt{2b \sinh\left(\frac{x}{2}\right)^4 + (a+b) \sinh\left(\frac{x}{2}\right)^2}} \right) \right. \\
& \left. \sqrt{\frac{2 \cosh\left(\frac{x}{2}\right)^2 b + a - b}{a-b}} \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2a+2b}{b}}\right) \right) \\
& + \frac{1}{b \sqrt{-\frac{2b}{a-b}} \sinh\left(\frac{x}{2}\right)^2 \left(2 \sinh\left(\frac{x}{2}\right)^2 b + a + b\right) (a^2 - b^2)} \left(2(Ab - aB) \sqrt{2b \sinh\left(\frac{x}{2}\right)^4 + (a+b) \sinh\left(\frac{x}{2}\right)^2} \right. \\
& - 2 \sqrt{-\frac{2b}{a-b}} b \cosh\left(\frac{x}{2}\right) \sinh\left(\frac{x}{2}\right)^2 + \operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2a+2b}{b}}\right) \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \sqrt{\frac{2b \sinh\left(\frac{x}{2}\right)^2}{a-b} + \frac{a+b}{a-b}} \\
& + \operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{-\frac{2b}{a-b}}, \sqrt{\frac{-2a+2b}{b}}\right) \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \sqrt{\frac{2b \sinh\left(\frac{x}{2}\right)^2}{a-b} + \frac{a+b}{a-b}} b - 2 \operatorname{EllipticE}\left(\cosh\left(\frac{x}{2}\right) \sqrt{-\frac{2b}{a-b}}, \right. \\
& \left. \sqrt{\frac{-2a+2b}{b}}\right) \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \sqrt{\frac{2b \sinh\left(\frac{x}{2}\right)^2}{a-b} + \frac{a+b}{a-b}} b \left. \right)
\end{aligned}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \cosh(x)}{(a + b \cosh(x))^5 / 2} dx$$

Optimal(type 4, 247 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{2(Ab - aB) \sinh(x)}{3(a^2 - b^2)(a + b \cosh(x))^3 \sqrt{2}} - \frac{2(4Aab - a^2B - 3Bb^2) \sinh(x)}{3(a^2 - b^2)^2 \sqrt{a + b \cosh(x)}} \\
 & - \frac{2I(4Aab - a^2B - 3Bb^2) \sqrt{\cosh\left(\frac{x}{2}\right)^2} \text{EllipticE}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{a + b \cosh(x)}}{3 \cosh\left(\frac{x}{2}\right) b (a^2 - b^2)^2 \sqrt{\frac{a + b \cosh(x)}{a+b}}} \\
 & + \frac{2I(Ab - aB) \sqrt{\cosh\left(\frac{x}{2}\right)^2} \text{EllipticF}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2} \sqrt{\frac{b}{a+b}}\right) \sqrt{\frac{a + b \cosh(x)}{a+b}}}{3 \cosh\left(\frac{x}{2}\right) b (a^2 - b^2) \sqrt{a + b \cosh(x)}}
 \end{aligned}$$

Result(type 4, 794 leaves):

$$\frac{1}{\sinh\left(\frac{x}{2}\right) \sqrt{2 \sinh\left(\frac{x}{2}\right)^2 b + a + b}} \left(\sqrt{\left(2 \cosh\left(\frac{x}{2}\right)^2 b + a - b\right) \sinh\left(\frac{x}{2}\right)^2} \right) \left(\frac{1}{b \sqrt{-\frac{2b}{a-b}} \sinh\left(\frac{x}{2}\right)^2 \left(2 \sinh\left(\frac{x}{2}\right)^2 b + a + b\right) (a^2 - b^2)} \right) \left(2B \right)$$

$$\sqrt{2b \sinh\left(\frac{x}{2}\right)^4 + (a+b) \sinh\left(\frac{x}{2}\right)^2} \left(-2 \sqrt{-\frac{2b}{a-b}} b \cosh\left(\frac{x}{2}\right) \sinh\left(\frac{x}{2}\right)^2 + \text{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{-\frac{2b}{a-b}}, \right. \right.$$

$$\left. \left. \sqrt{\frac{-2a+2b}{b}} \right) \sqrt{-\sinh\left(\frac{x}{2}\right)^2} \sqrt{\frac{2b \sinh\left(\frac{x}{2}\right)^2}{a-b} + \frac{a+b}{a-b}} a + \text{EllipticF}\left(\cosh\left(\frac{x}{2}\right) \sqrt{-\frac{2b}{a-b}}, \right. \right.$$

$$\frac{\sqrt{a \sinh(x)^2} \left(-\ln \left(\frac{2 (\sqrt{-a} \sqrt{a \sinh(x)^2 - a})}{\cosh(x)} \right) a \cosh(x)^2 + \sqrt{-a} \sqrt{a \sinh(x)^2} \right)}{2 a^2 \cosh(x) \sqrt{-a} \sinh(x) \sqrt{a \cosh(x)^2}}$$

Problem 39: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a \cosh(x)^3}} dx$$

Optimal(type 4, 55 leaves, 3 steps):

$$\frac{2 I \cosh(x)^3 /2 \sqrt{\cosh\left(\frac{x}{2}\right)^2} \text{EllipticE}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{x}{2}\right) \sqrt{a \cosh(x)^3}} + \frac{2 \cosh(x) \sinh(x)}{\sqrt{a \cosh(x)^3}}$$

Result(type 8, 10 leaves):

$$\int \frac{1}{\sqrt{a \cosh(x)^3}} dx$$

Problem 40: Unable to integrate problem.

$$\int \frac{1}{(a \cosh(x)^3)^{5/2}} dx$$

Optimal(type 4, 114 leaves, 6 steps):

$$\frac{154 I \cosh(x)^3 /2 \sqrt{\cosh\left(\frac{x}{2}\right)^2} \text{EllipticE}\left(I \sinh\left(\frac{x}{2}\right), \sqrt{2}\right)}{195 \cosh\left(\frac{x}{2}\right) a^2 \sqrt{a \cosh(x)^3}} + \frac{154 \cosh(x) \sinh(x)}{195 a^2 \sqrt{a \cosh(x)^3}} + \frac{154 \tanh(x)}{585 a^2 \sqrt{a \cosh(x)^3}} + \frac{22 \operatorname{sech}(x)^2 \tanh(x)}{117 a^2 \sqrt{a \cosh(x)^3}} + \frac{2 \operatorname{sech}(x)^4 \tanh(x)}{13 a^2 \sqrt{a \cosh(x)^3}}$$

Result(type 8, 10 leaves):

$$\int \frac{1}{(a \cosh(x)^3)^{5/2}} dx$$

Problem 44: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^8}{a + a \cosh(x)} dx$$

Optimal(type 3, 47 leaves, 5 steps):

$$\frac{5x}{16a} - \frac{5 \cosh(x) \sinh(x)}{16a} + \frac{5 \cosh(x) \sinh(x)^3}{24a} - \frac{\cosh(x) \sinh(x)^5}{6a} + \frac{\sinh(x)^7}{7a}$$

Result(type 3, 207 leaves):

$$\begin{aligned}
 & -\frac{1}{7a \left(\tanh\left(\frac{x}{2}\right) + 1\right)^7} + \frac{2}{3a \left(\tanh\left(\frac{x}{2}\right) + 1\right)^6} - \frac{1}{a \left(\tanh\left(\frac{x}{2}\right) + 1\right)^5} + \frac{1}{4a \left(\tanh\left(\frac{x}{2}\right) + 1\right)^4} + \frac{11}{24a \left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} + \frac{1}{8a \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} \\
 & - \frac{5}{16a \left(\tanh\left(\frac{x}{2}\right) + 1\right)} + \frac{5 \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{16a} - \frac{1}{7a \left(\tanh\left(\frac{x}{2}\right) - 1\right)^7} - \frac{2}{3a \left(\tanh\left(\frac{x}{2}\right) - 1\right)^6} - \frac{1}{a \left(\tanh\left(\frac{x}{2}\right) - 1\right)^5} \\
 & - \frac{1}{4a \left(\tanh\left(\frac{x}{2}\right) - 1\right)^4} + \frac{11}{24a \left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} - \frac{1}{8a \left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} - \frac{5}{16a \left(\tanh\left(\frac{x}{2}\right) - 1\right)} - \frac{5 \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{16a}
 \end{aligned}$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^5}{a + a \cosh(x)} dx$$

Optimal(type 3, 29 leaves, 3 steps):

$$-\frac{2(a - a \cosh(x))^3}{3a^4} + \frac{(a - a \cosh(x))^4}{4a^5}$$

Result(type 3, 86 leaves):

$$\begin{aligned}
 & \frac{1}{a} \left(32 \left(\frac{1}{128 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^4} - \frac{5}{192 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} + \frac{5}{256 \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} + \frac{5}{256 \left(\tanh\left(\frac{x}{2}\right) + 1\right)} + \frac{1}{128 \left(\tanh\left(\frac{x}{2}\right) - 1\right)^4} \right. \right. \\
 & \left. \left. + \frac{5}{192 \left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} + \frac{5}{256 \left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} - \frac{5}{256 \left(\tanh\left(\frac{x}{2}\right) - 1\right)} \right) \right)
 \end{aligned}$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^4}{a + a \cosh(x)} dx$$

Optimal(type 3, 25 leaves, 3 steps):

$$\frac{x}{2a} - \frac{\cosh(x) \sinh(x)}{2a} + \frac{\sinh(x)^3}{3a}$$

Result(type 3, 102 leaves):

$$\begin{aligned}
& -\frac{1}{3a \left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} + \frac{1}{a \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} - \frac{1}{2a \left(\tanh\left(\frac{x}{2}\right) + 1\right)} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{2a} - \frac{1}{3a \left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} - \frac{1}{a \left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} \\
& - \frac{1}{2a \left(\tanh\left(\frac{x}{2}\right) - 1\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{2a}
\end{aligned}$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^7}{a + b \cosh(x)} dx$$

Optimal (type 3, 130 leaves, 3 steps):

$$\begin{aligned}
& -\frac{a(a^4 - 3a^2b^2 + 3b^4) \cosh(x)}{b^6} + \frac{(a^4 - 3a^2b^2 + 3b^4) \cosh(x)^2}{2b^5} - \frac{a(a^2 - 3b^2) \cosh(x)^3}{3b^4} + \frac{(a^2 - 3b^2) \cosh(x)^4}{4b^3} - \frac{a \cosh(x)^5}{5b^2} + \frac{\cosh(x)^6}{6b} \\
& + \frac{(a^2 - b^2)^3 \ln(a + b \cosh(x))}{b^7}
\end{aligned}$$

Result (type 3, 1038 leaves):

$$\begin{aligned}
& \frac{5}{16b \left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} + \frac{1}{6b \left(\tanh\left(\frac{x}{2}\right) + 1\right)^6} + \frac{1}{6b \left(\tanh\left(\frac{x}{2}\right) - 1\right)^6} + \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right)}{a - b} - \frac{1}{2b \left(\tanh\left(\frac{x}{2}\right) + 1\right)^5} \\
& + \frac{1}{8b \left(\tanh\left(\frac{x}{2}\right) + 1\right)^4} + \frac{7}{12b \left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{b} + \frac{5}{16b \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} + \frac{1}{2b \left(\tanh\left(\frac{x}{2}\right) - 1\right)^5} \\
& + \frac{1}{8b \left(\tanh\left(\frac{x}{2}\right) - 1\right)^4} - \frac{7}{12b \left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b} - \frac{11}{16b \left(\tanh\left(\frac{x}{2}\right) + 1\right)} + \frac{11}{16b \left(\tanh\left(\frac{x}{2}\right) - 1\right)} \\
& + \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right) a^7}{b^7 (a - b)} - \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right) a^6}{b^6 (a - b)} - \frac{3 \ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right) a^5}{b^5 (a - b)} \\
& + \frac{3 \ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right) a^4}{b^4 (a - b)} + \frac{3 \ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right) a^3}{b^3 (a - b)} - \frac{3 \ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right) a^2}{b^2 (a - b)}
\end{aligned}$$

$$\begin{aligned}
& - \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right) a}{b(a-b)} - \frac{7a^2}{8b^3\left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} - \frac{7a}{8b^2\left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} - \frac{a^5}{b^6\left(\tanh\left(\frac{x}{2}\right) + 1\right)} - \frac{a^4}{2b^5\left(\tanh\left(\frac{x}{2}\right) + 1\right)} \\
& + \frac{5a^3}{2b^4\left(\tanh\left(\frac{x}{2}\right) + 1\right)} + \frac{9a^2}{8b^3\left(\tanh\left(\frac{x}{2}\right) + 1\right)} - \frac{15a}{8b^2\left(\tanh\left(\frac{x}{2}\right) + 1\right)} + \frac{a}{5b^2\left(\tanh\left(\frac{x}{2}\right) - 1\right)^5} + \frac{a^2}{4b^3\left(\tanh\left(\frac{x}{2}\right) - 1\right)^4} \\
& + \frac{a}{2b^2\left(\tanh\left(\frac{x}{2}\right) - 1\right)^4} - \frac{a}{5b^2\left(\tanh\left(\frac{x}{2}\right) + 1\right)^5} + \frac{a^3}{3b^4\left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} + \frac{a^2}{2b^3\left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} - \frac{a}{4b^2\left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} \\
& - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) a^6}{b^7} + \frac{3\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) a^4}{b^5} - \frac{3\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) a^2}{b^3} + \frac{a^4}{2b^5\left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} + \frac{a^3}{2b^4\left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} \\
& - \frac{7a^2}{8b^3\left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} - \frac{7a}{8b^2\left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} + \frac{a^5}{b^6\left(\tanh\left(\frac{x}{2}\right) - 1\right)} + \frac{a^4}{2b^5\left(\tanh\left(\frac{x}{2}\right) - 1\right)} - \frac{5a^3}{2b^4\left(\tanh\left(\frac{x}{2}\right) - 1\right)} \\
& - \frac{9a^2}{8b^3\left(\tanh\left(\frac{x}{2}\right) - 1\right)} + \frac{15a}{8b^2\left(\tanh\left(\frac{x}{2}\right) - 1\right)} + \frac{a^2}{4b^3\left(\tanh\left(\frac{x}{2}\right) + 1\right)^4} + \frac{a}{2b^2\left(\tanh\left(\frac{x}{2}\right) + 1\right)^4} - \frac{a^3}{3b^4\left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} \\
& - \frac{a^2}{2b^3\left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} + \frac{a}{4b^2\left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) a^6}{b^7} + \frac{3\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) a^4}{b^5} - \frac{3\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) a^2}{b^3} \\
& + \frac{a^4}{2b^5\left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} + \frac{a^3}{2b^4\left(\tanh\left(\frac{x}{2}\right) + 1\right)^2}
\end{aligned}$$

Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^5}{a + b \cosh(x)} dx$$

Optimal(type 3, 77 leaves, 3 steps):

$$-\frac{a(a^2 - 2b^2) \cosh(x)}{b^4} + \frac{(a^2 - 2b^2) \cosh(x)^2}{2b^3} - \frac{a \cosh(x)^3}{3b^2} + \frac{\cosh(x)^4}{4b} + \frac{(a^2 - b^2)^2 \ln(a + b \cosh(x))}{b^5}$$

Result(type 3, 598 leaves):

$$\begin{aligned}
& -\frac{3}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{\ln\left(a\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right)}{a-b} + \frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)+1\right)^4} - \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^3} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b} \\
& - \frac{3}{8b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{1}{4b\left(\tanh\left(\frac{x}{2}\right)-1\right)^4} + \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b} + \frac{5}{8b\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{5}{8b\left(\tanh\left(\frac{x}{2}\right)-1\right)} \\
& + \frac{\ln\left(a\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right)a^5}{b^5(a-b)} - \frac{\ln\left(a\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right)a^4}{b^4(a-b)} - \frac{2\ln\left(a\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right)a^3}{b^3(a-b)} \\
& + \frac{2\ln\left(a\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right)a^2}{b^2(a-b)} + \frac{\ln\left(a\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - a - b\right)a}{b(a-b)} + \frac{a^2}{2b^3\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{a}{2b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} \\
& - \frac{a^3}{b^4\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{a^2}{2b^3\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{3a}{2b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{a}{3b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)a^4}{b^5} \\
& + \frac{2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)a^2}{b^3} + \frac{a^2}{2b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{a^3}{b^4\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{a^2}{2b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)} \\
& - \frac{3a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)} - \frac{a}{3b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^3} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)a^4}{b^5} + \frac{2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)a^2}{b^3}
\end{aligned}$$

Problem 60: Unable to integrate problem.

$$\int \frac{x^3 \sinh(dx+c)^2}{a+b \cosh(dx+c)} dx$$

Optimal (type 4, 453 leaves, 18 steps):

$$-\frac{ax^4}{4b^2} - \frac{6 \cosh(dx+c)}{bd^4} - \frac{3x^2 \cosh(dx+c)}{bd^2} + \frac{6x \sinh(dx+c)}{bd^3} + \frac{x^3 \sinh(dx+c)}{bd} + \frac{x^3 \ln\left(1 + \frac{be^{dx+c}}{a - \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d}$$

$$\begin{aligned}
& - \frac{x^3 \ln\left(1 + \frac{b e^{dx+c}}{a + \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d} + \frac{3 x^2 \operatorname{polylog}\left(2, -\frac{b e^{dx+c}}{a - \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d^2} - \frac{3 x^2 \operatorname{polylog}\left(2, -\frac{b e^{dx+c}}{a + \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d^2} \\
& - \frac{6 x \operatorname{polylog}\left(3, -\frac{b e^{dx+c}}{a - \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d^3} + \frac{6 x \operatorname{polylog}\left(3, -\frac{b e^{dx+c}}{a + \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d^3} + \frac{6 \operatorname{polylog}\left(4, -\frac{b e^{dx+c}}{a - \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d^4} \\
& - \frac{6 \operatorname{polylog}\left(4, -\frac{b e^{dx+c}}{a + \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d^4}
\end{aligned}$$

Result(type 8, 130 leaves):

$$-\frac{ax^4}{4b^2} + \frac{(x^3 d^3 - 3d^2 x^2 + 6dx - 6) e^{dx+c}}{2d^4 b} - \frac{x^3 d^3 + 3d^2 x^2 + 6dx + 6}{2d^4 b e^{dx+c}} + \int \frac{2x^3 (a^2 - b^2) e^{dx+c}}{(b (e^{dx+c})^2 + 2a e^{dx+c} + b) b^2} dx$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \frac{x \sinh(dx+c)^2}{a + b \cosh(dx+c)} dx$$

Optimal(type 4, 222 leaves, 12 steps):

$$\begin{aligned}
& - \frac{ax^2}{2b^2} - \frac{\cosh(dx+c)}{bd^2} + \frac{x \sinh(dx+c)}{bd} + \frac{x \ln\left(1 + \frac{b e^{dx+c}}{a - \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d} - \frac{x \ln\left(1 + \frac{b e^{dx+c}}{a + \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d} \\
& + \frac{\operatorname{polylog}\left(2, -\frac{b e^{dx+c}}{a - \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d^2} - \frac{\operatorname{polylog}\left(2, -\frac{b e^{dx+c}}{a + \sqrt{a^2 - b^2}}\right) \sqrt{a^2 - b^2}}{b^2 d^2}
\end{aligned}$$

Result(type 4, 861 leaves):

$$\begin{aligned}
& - \frac{ax^2}{2b^2} + \frac{(dx-1) e^{dx+c}}{2bd^2} - \frac{(dx+1) e^{-dx-c}}{2bd^2} + \frac{\ln\left(\frac{-b e^{dx+c} + \sqrt{a^2 - b^2} - a}{-a + \sqrt{a^2 - b^2}}\right) x a^2}{b^2 d \sqrt{a^2 - b^2}} - \frac{\ln\left(\frac{-b e^{dx+c} + \sqrt{a^2 - b^2} - a}{-a + \sqrt{a^2 - b^2}}\right) x}{d \sqrt{a^2 - b^2}} \\
& - \frac{\ln\left(\frac{b e^{dx+c} + \sqrt{a^2 - b^2} + a}{a + \sqrt{a^2 - b^2}}\right) x a^2}{b^2 d \sqrt{a^2 - b^2}} + \frac{\ln\left(\frac{b e^{dx+c} + \sqrt{a^2 - b^2} + a}{a + \sqrt{a^2 - b^2}}\right) x}{d \sqrt{a^2 - b^2}} + \frac{\ln\left(\frac{-b e^{dx+c} + \sqrt{a^2 - b^2} - a}{-a + \sqrt{a^2 - b^2}}\right) c a^2}{b^2 d^2 \sqrt{a^2 - b^2}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{\ln\left(\frac{-b e^{dx+c} + \sqrt{a^2 - b^2} - a}{-a + \sqrt{a^2 - b^2}}\right) c}{d^2 \sqrt{a^2 - b^2}} - \frac{\ln\left(\frac{b e^{dx+c} + \sqrt{a^2 - b^2} + a}{a + \sqrt{a^2 - b^2}}\right) c a^2}{b^2 d^2 \sqrt{a^2 - b^2}} + \frac{\ln\left(\frac{b e^{dx+c} + \sqrt{a^2 - b^2} + a}{a + \sqrt{a^2 - b^2}}\right) c}{d^2 \sqrt{a^2 - b^2}} \\
& + \frac{\operatorname{dilog}\left(\frac{-b e^{dx+c} + \sqrt{a^2 - b^2} - a}{-a + \sqrt{a^2 - b^2}}\right) a^2}{b^2 d^2 \sqrt{a^2 - b^2}} - \frac{\operatorname{dilog}\left(\frac{-b e^{dx+c} + \sqrt{a^2 - b^2} - a}{-a + \sqrt{a^2 - b^2}}\right)}{d^2 \sqrt{a^2 - b^2}} - \frac{\operatorname{dilog}\left(\frac{b e^{dx+c} + \sqrt{a^2 - b^2} + a}{a + \sqrt{a^2 - b^2}}\right) a^2}{b^2 d^2 \sqrt{a^2 - b^2}} \\
& + \frac{\operatorname{dilog}\left(\frac{b e^{dx+c} + \sqrt{a^2 - b^2} + a}{a + \sqrt{a^2 - b^2}}\right)}{d^2 \sqrt{a^2 - b^2}} - \frac{2 c \arctan\left(\frac{2 b e^{dx+c} + 2 a}{2 \sqrt{-a^2 + b^2}}\right) a^2}{b^2 d^2 \sqrt{-a^2 + b^2}} + \frac{2 c \arctan\left(\frac{2 b e^{dx+c} + 2 a}{2 \sqrt{-a^2 + b^2}}\right)}{d^2 \sqrt{-a^2 + b^2}}
\end{aligned}$$

Problem 62: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(dx+c)^2}{a+b \cosh(dx+c)} dx$$

Optimal (type 3, 64 leaves, 4 steps):

$$-\frac{ax}{b^2} + \frac{\sinh(dx+c)}{bd} + \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a-b} \tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{\sqrt{a+b}}\right) \sqrt{a-b} \sqrt{a+b}}{b^2 d}$$

Result (type 3, 176 leaves):

$$\begin{aligned}
& \frac{2 \operatorname{arctanh}\left(\frac{(a-b) \tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{\sqrt{(a+b)(a-b)}}\right) a^2}{db^2 \sqrt{(a+b)(a-b)}} - \frac{2 \operatorname{arctanh}\left(\frac{(a-b) \tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d \sqrt{(a+b)(a-b)}} - \frac{1}{db \left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right) + 1\right)} - \frac{a \ln\left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right) + 1\right)}{db^2} \\
& - \frac{1}{db \left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right) - 1\right)} + \frac{a \ln\left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right) - 1\right)}{db^2}
\end{aligned}$$

Problem 65: Unable to integrate problem.

$$\int \cosh(a + b \ln(cx^n)) dx$$

Optimal (type 3, 54 leaves, 1 step):

$$\frac{x \cosh(a + b \ln(cx^n))}{-b^2 n^2 + 1} - \frac{b n x \sinh(a + b \ln(cx^n))}{-b^2 n^2 + 1}$$

Result(type 8, 13 leaves):

$$\int \cosh(a + b \ln(cx^n)) dx$$

Problem 66: Unable to integrate problem.

$$\int \cosh(a + b \ln(cx^n))^4 dx$$

Optimal(type 3, 191 leaves, 3 steps):

$$\frac{24 b^4 n^4 x}{64 b^4 n^4 - 20 b^2 n^2 + 1} - \frac{12 b^2 n^2 x \cosh(a + b \ln(cx^n))^2}{64 b^4 n^4 - 20 b^2 n^2 + 1} + \frac{x \cosh(a + b \ln(cx^n))^4}{-16 b^2 n^2 + 1} + \frac{24 b^3 n^3 x \cosh(a + b \ln(cx^n)) \sinh(a + b \ln(cx^n))}{64 b^4 n^4 - 20 b^2 n^2 + 1} - \frac{4 b n x \cosh(a + b \ln(cx^n))^3 \sinh(a + b \ln(cx^n))}{-16 b^2 n^2 + 1}$$

Result(type 8, 15 leaves):

$$\int \cosh(a + b \ln(cx^n))^4 dx$$

Problem 67: Unable to integrate problem.

$$\int x^m \cosh(a + b \ln(cx^n)) dx$$

Optimal(type 3, 73 leaves, 1 step):

$$\frac{(1 + m) x^{1+m} \cosh(a + b \ln(cx^n))}{(1 + m)^2 - b^2 n^2} - \frac{b n x^{1+m} \sinh(a + b \ln(cx^n))}{(1 + m)^2 - b^2 n^2}$$

Result(type 8, 17 leaves):

$$\int x^m \cosh(a + b \ln(cx^n)) dx$$

Problem 70: Unable to integrate problem.

$$\int \cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)^{5/2} dx$$

Optimal(type 3, 202 leaves, 8 steps):

$$\begin{aligned}
& -\frac{x \cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)^{5/2}}{4} + \frac{5x \cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)^{5/2}}{4 e^{2a} (cx^n)^{\frac{4}{n}} \left(1 + \frac{1}{e^{2a} (cx^n)^{\frac{4}{n}}}\right)^2} + \frac{5x \cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)^{5/2}}{12 \left(1 + \frac{1}{e^{2a} (cx^n)^{\frac{4}{n}}}\right)} \\
& - \frac{5x \operatorname{arccsch}\left(e^a (cx^n)^{\frac{2}{n}}\right) \cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)^{5/2}}{4 e^{3a} (cx^n)^{\frac{6}{n}} \left(1 + \frac{1}{e^{2a} (cx^n)^{\frac{4}{n}}}\right)^{5/2}}
\end{aligned}$$

Result(type 8, 18 leaves):

$$\int \cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)^{5/2} dx$$

Problem 71: Unable to integrate problem.

$$\int \sqrt{\cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)} dx$$

Optimal(type 3, 95 leaves, 6 steps):

$$\frac{x \sqrt{\cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)}}{2} - \frac{x \operatorname{arccsch}\left(e^a (cx^n)^{\frac{2}{n}}\right) \sqrt{\cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)}}{2 e^a (cx^n)^{\frac{2}{n}} \sqrt{1 + \frac{1}{e^{2a} (cx^n)^{\frac{4}{n}}}}}$$

Result(type 8, 18 leaves):

$$\int \sqrt{\cosh\left(a + \frac{2 \ln(cx^n)}{n}\right)} dx$$

Problem 77: Result is not expressed in closed-form.

$$\int e^x \operatorname{sech}(4x)^2 dx$$

Optimal(type 3, 263 leaves, 22 steps):

$$\begin{aligned}
& -\frac{e^x}{2(1+e^{8x})} - \frac{\ln\left(1+e^{2x}-e^x\sqrt{2-\sqrt{2}}\right)\sqrt{2-\sqrt{2}}}{32} + \frac{\ln\left(1+e^{2x}+e^x\sqrt{2-\sqrt{2}}\right)\sqrt{2-\sqrt{2}}}{32} - \frac{\arctan\left(\frac{-2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{4-2\sqrt{2}}} \\
& + \frac{\arctan\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{4-2\sqrt{2}}} - \frac{\ln\left(1+e^{2x}-e^x\sqrt{2+\sqrt{2}}\right)\sqrt{2+\sqrt{2}}}{32} + \frac{\ln\left(1+e^{2x}+e^x\sqrt{2+\sqrt{2}}\right)\sqrt{2+\sqrt{2}}}{32} - \frac{\arctan\left(\frac{-2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{4+2\sqrt{2}}} \\
& + \frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{4+2\sqrt{2}}}
\end{aligned}$$

Result(type 7, 35 leaves):

$$-\frac{e^x}{2(1+e^{8x})} + 4 \left(\sum_{R=\text{RootOf}(281474976710656z^8+1)} -R \ln(e^x + 64_R) \right)$$

Problem 78: Unable to integrate problem.

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^2 dx$$

Optimal(type 5, 68 leaves, 1 step):

$$\frac{4e^{2ex+2d} F^{c(bx+a)} \operatorname{hypergeom}\left(\left[2, 1 + \frac{bc \ln(F)}{2e}\right], \left[2 + \frac{bc \ln(F)}{2e}\right], -e^{2ex+2d}\right)}{bc \ln(F) + 2e}$$

Result(type 8, 20 leaves):

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^2 dx$$

Problem 86: Unable to integrate problem.

$$\int \left(\frac{x}{\cosh(x)^{5/2}} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx$$

Optimal(type 3, 16 leaves, 2 steps):

$$\frac{2x \sinh(x)}{3 \cosh(x)^{3/2}} + \frac{4}{3\sqrt{\cosh(x)}}$$

Result(type 8, 16 leaves):

$$\int \left(\frac{x}{\cosh(x)^{5/2}} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx$$

Problem 87: Unable to integrate problem.

$$\int \left(\frac{x^2}{\cosh(x)^3 \sqrt{2}} + x^2 \sqrt{\cosh(x)} \right) dx$$

Optimal(type 4, 47 leaves, 3 steps):

$$-\frac{16 \sqrt{\cosh\left(\frac{x}{2}\right)^2} \operatorname{EllipticE}\left(\operatorname{I} \sinh\left(\frac{x}{2}\right), \sqrt{2}\right)}{\cosh\left(\frac{x}{2}\right)} + \frac{2x^2 \sinh(x)}{\sqrt{\cosh(x)}} - 8x \sqrt{\cosh(x)}$$

Result(type 8, 19 leaves):

$$\int \left(\frac{x^2}{\cosh(x)^3 \sqrt{2}} + x^2 \sqrt{\cosh(x)} \right) dx$$

Test results for the 25 problems in "6.2.7 hyper^m (a+b cosh^n)^p.txt"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^4}{a - a \cosh(x)^2} dx$$

Optimal(type 3, 16 leaves, 3 steps):

$$\frac{x}{2a} - \frac{\cosh(x) \sinh(x)}{2a}$$

Result(type 3, 77 leaves):

$$\frac{1}{2a \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^2} - \frac{1}{2a \left(\tanh\left(\frac{x}{2}\right) + 1 \right)} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{2a} - \frac{1}{2a \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^2} - \frac{1}{2a \left(\tanh\left(\frac{x}{2}\right) - 1 \right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{2a}$$

Problem 2: Result unnecessarily involves higher level functions.

$$\int \frac{\sinh(x)^2}{a - a \cosh(x)^2} dx$$

Optimal(type 1, 6 leaves, 2 steps):

$$-\frac{x}{a}$$

Result(type 3, 10 leaves):

$$-\frac{2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^5}{a + b \cosh(x)^2} dx$$

Optimal(type 3, 44 leaves, 4 steps):

$$-\frac{(a+2b)\cosh(x)}{b^2} + \frac{\cosh(x)^3}{3b} + \frac{(a+b)^2 \arctan\left(\frac{\cosh(x)\sqrt{b}}{\sqrt{a}}\right)}{b^5/2\sqrt{a}}$$

Result(type 3, 213 leaves):

$$\begin{aligned} & \frac{\arctan\left(\frac{2(a+b)\tanh\left(\frac{x}{2}\right)^2 - 2a + 2b}{4\sqrt{ab}}\right) a^2}{b^2\sqrt{ab}} + \frac{2\arctan\left(\frac{2(a+b)\tanh\left(\frac{x}{2}\right)^2 - 2a + 2b}{4\sqrt{ab}}\right) a}{b\sqrt{ab}} + \frac{\arctan\left(\frac{2(a+b)\tanh\left(\frac{x}{2}\right)^2 - 2a + 2b}{4\sqrt{ab}}\right)}{\sqrt{ab}} \\ & + \frac{1}{3b\left(\tanh\left(\frac{x}{2}\right) + 1\right)^3} - \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} - \frac{a}{b^2\left(\tanh\left(\frac{x}{2}\right) + 1\right)} - \frac{3}{2b\left(\tanh\left(\frac{x}{2}\right) + 1\right)} - \frac{1}{3b\left(\tanh\left(\frac{x}{2}\right) - 1\right)^3} \\ & - \frac{1}{2b\left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} + \frac{a}{b^2\left(\tanh\left(\frac{x}{2}\right) - 1\right)} + \frac{3}{2b\left(\tanh\left(\frac{x}{2}\right) - 1\right)} \end{aligned}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^2}{a + b \cosh(x)^2} dx$$

Optimal(type 3, 31 leaves, 4 steps):

$$\frac{x}{b} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a+b}}\right)\sqrt{a+b}}{b\sqrt{a}}$$

Result(type 3, 188 leaves):

$$\begin{aligned} & \frac{\sqrt{a}\ln\left(-\sqrt{a+b}\tanh\left(\frac{x}{2}\right)^2 + 2\tanh\left(\frac{x}{2}\right)\sqrt{a} - \sqrt{a+b}\right)}{2b\sqrt{a+b}} - \frac{\sqrt{a}\ln\left(\sqrt{a+b}\tanh\left(\frac{x}{2}\right)^2 + 2\tanh\left(\frac{x}{2}\right)\sqrt{a} + \sqrt{a+b}\right)}{2b\sqrt{a+b}} \\ & + \frac{\ln\left(-\sqrt{a+b}\tanh\left(\frac{x}{2}\right)^2 + 2\tanh\left(\frac{x}{2}\right)\sqrt{a} - \sqrt{a+b}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{\ln\left(\sqrt{a+b}\tanh\left(\frac{x}{2}\right)^2 + 2\tanh\left(\frac{x}{2}\right)\sqrt{a} + \sqrt{a+b}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{b} \\ & - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b} \end{aligned}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{a + b \cosh(x)^2} dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b}}\right)}{\sqrt{a} \sqrt{a+b}}$$

Result (type 3, 80 leaves):

$$-\frac{\ln\left(-\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} - \sqrt{a+b}\right)}{2\sqrt{a} \sqrt{a+b}} + \frac{\ln\left(\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} + \sqrt{a+b}\right)}{2\sqrt{a} \sqrt{a+b}}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{csch}(x)^4}{a + b \cosh(x)^2} dx$$

Optimal (type 3, 49 leaves, 4 steps):

$$\frac{(a+2b) \operatorname{coth}(x)}{(a+b)^2} - \frac{\operatorname{coth}(x)^3}{3(a+b)} + \frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b}}\right)}{(a+b)^5 / 2\sqrt{a}}$$

Result (type 3, 179 leaves):

$$-\frac{\tanh\left(\frac{x}{2}\right)^3 a}{24(a+b)^2} - \frac{\tanh\left(\frac{x}{2}\right)^3 b}{24(a+b)^2} + \frac{3 \tanh\left(\frac{x}{2}\right) a}{8(a+b)^2} + \frac{7 \tanh\left(\frac{x}{2}\right) b}{8(a+b)^2} - \frac{1}{24(a+b) \tanh\left(\frac{x}{2}\right)^3} + \frac{3a}{8(a+b)^2 \tanh\left(\frac{x}{2}\right)} + \frac{7b}{8(a+b)^2 \tanh\left(\frac{x}{2}\right)}$$

$$-\frac{b^2 \ln\left(-\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} - \sqrt{a+b}\right)}{2(a+b)^5 / 2\sqrt{a}} + \frac{b^2 \ln\left(\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} + \sqrt{a+b}\right)}{2(a+b)^5 / 2\sqrt{a}}$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{csch}(x)^6}{a + b \cosh(x)^2} dx$$

Optimal (type 3, 77 leaves, 4 steps):

$$-\frac{(a^2 + 3ab + 3b^2) \coth(x)}{(a+b)^3} + \frac{(2a+3b) \coth(x)^3}{3(a+b)^2} - \frac{\coth(x)^5}{5(a+b)} - \frac{b^3 \operatorname{arctanh}\left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b}}\right)}{(a+b)^{7/2} \sqrt{a}}$$

Result(type 3, 309 leaves):

$$\begin{aligned} & -\frac{\tanh\left(\frac{x}{2}\right)^5 a^2}{160(a+b)^3} - \frac{\tanh\left(\frac{x}{2}\right)^5 ab}{80(a+b)^3} - \frac{\tanh\left(\frac{x}{2}\right)^5 b^2}{160(a+b)^3} + \frac{5 \tanh\left(\frac{x}{2}\right)^3 a^2}{96(a+b)^3} + \frac{7 \tanh\left(\frac{x}{2}\right)^3 ab}{48(a+b)^3} + \frac{3 \tanh\left(\frac{x}{2}\right)^3 b^2}{32(a+b)^3} - \frac{5 \tanh\left(\frac{x}{2}\right) a^2}{16(a+b)^3} - \frac{\tanh\left(\frac{x}{2}\right) ab}{(a+b)^3} \\ & - \frac{19 \tanh\left(\frac{x}{2}\right) b^2}{16(a+b)^3} - \frac{1}{160(a+b) \tanh\left(\frac{x}{2}\right)^5} + \frac{5a}{96(a+b)^2 \tanh\left(\frac{x}{2}\right)^3} + \frac{3b}{32(a+b)^2 \tanh\left(\frac{x}{2}\right)^3} - \frac{5a^2}{16(a+b)^3 \tanh\left(\frac{x}{2}\right)} - \frac{ab}{(a+b)^3 \tanh\left(\frac{x}{2}\right)} \\ & - \frac{19b^2}{16(a+b)^3 \tanh\left(\frac{x}{2}\right)} + \frac{b^3 \ln\left(-\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} - \sqrt{a+b}\right)}{2(a+b)^{7/2} \sqrt{a}} - \frac{b^3 \ln\left(\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} + \sqrt{a+b}\right)}{2(a+b)^{7/2} \sqrt{a}} \end{aligned}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(x)^4}{a+b \cosh(x)^2} dx$$

Optimal(type 3, 47 leaves, 5 steps):

$$-\frac{(2a-b)x}{2b^2} + \frac{\cosh(x) \sinh(x)}{2b} + \frac{a^{3/2} \operatorname{arctanh}\left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b}}\right)}{b^2 \sqrt{a+b}}$$

Result(type 3, 187 leaves):

$$\begin{aligned} & -\frac{a^{3/2} \ln\left(-\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} - \sqrt{a+b}\right)}{2b^2 \sqrt{a+b}} + \frac{a^{3/2} \ln\left(\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} + \sqrt{a+b}\right)}{2b^2 \sqrt{a+b}} - \frac{1}{2b \left(\tanh\left(\frac{x}{2}\right) + 1\right)^2} \\ & + \frac{1}{2b \left(\tanh\left(\frac{x}{2}\right) + 1\right)} - \frac{a \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{b^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) + 1\right)}{2b} + \frac{1}{2b \left(\tanh\left(\frac{x}{2}\right) - 1\right)^2} + \frac{1}{2b \left(\tanh\left(\frac{x}{2}\right) - 1\right)} + \frac{a \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b^2} \\ & - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{2b} \end{aligned}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^2}{a + b \cosh(x)^2} dx$$

Optimal(type 3, 30 leaves, 3 steps):

$$-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{a} \tanh(x)}{\sqrt{a+b}}\right)}{a^3 / 2 \sqrt{a+b}} + \frac{\tanh(x)}{a}$$

Result(type 3, 101 leaves):

$$\frac{b \ln\left(-\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} - \sqrt{a+b}\right)}{2 a^3 / 2 \sqrt{a+b}} - \frac{b \ln\left(\sqrt{a+b} \tanh\left(\frac{x}{2}\right)^2 + 2 \tanh\left(\frac{x}{2}\right) \sqrt{a} + \sqrt{a+b}\right)}{2 a^3 / 2 \sqrt{a+b}} + \frac{2 \tanh\left(\frac{x}{2}\right)}{a \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\cosh(x)^2 + 1} dx$$

Optimal(type 3, 13 leaves, 2 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\tanh(x) \sqrt{2}}{2}\right) \sqrt{2}}{2}$$

Result(type 3, 85 leaves):

$$\frac{\sqrt{2} \ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2 + \tanh\left(\frac{x}{2}\right) \sqrt{2} + 1}{\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right) \sqrt{2} + 1}\right)}{8} - \frac{\sqrt{2} \ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right) \sqrt{2} + 1}{\tanh\left(\frac{x}{2}\right)^2 + \tanh\left(\frac{x}{2}\right) \sqrt{2} + 1}\right)}{8}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(1 - \cosh(x)^2)^2} dx$$

Optimal(type 3, 9 leaves, 3 steps):

$$\operatorname{coth}(x) - \frac{\operatorname{coth}(x)^3}{3}$$

Result(type 3, 31 leaves):

$$-\frac{\tanh\left(\frac{x}{2}\right)^3}{24} + \frac{3 \tanh\left(\frac{x}{2}\right)}{8} - \frac{1}{24 \tanh\left(\frac{x}{2}\right)^3} + \frac{3}{8 \tanh\left(\frac{x}{2}\right)}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + b \cosh(x)^2} dx$$

Optimal(type 4, 47 leaves, 2 steps):

$$\frac{\sqrt{-\sinh(x)^2} \operatorname{EllipticE}\left(\cosh(x), \sqrt{-\frac{b}{a}}\right) \sqrt{a + b \cosh(x)^2}}{\sinh(x) \sqrt{1 + \frac{b \cosh(x)^2}{a}}}$$

Result(type 4, 113 leaves):

$$\frac{\sqrt{\frac{a + b \cosh(x)^2}{a}} \sqrt{-\sinh(x)^2} \left(a \operatorname{EllipticF}\left(\cosh(x) \sqrt{-\frac{b}{a}}, \sqrt{-\frac{a}{b}}\right) + b \operatorname{EllipticF}\left(\cosh(x) \sqrt{-\frac{b}{a}}, \sqrt{-\frac{a}{b}}\right) - b \operatorname{EllipticE}\left(\cosh(x) \sqrt{-\frac{b}{a}}, \sqrt{-\frac{a}{b}}\right) \right)}{\sqrt{-\frac{b}{a}} \sinh(x) \sqrt{a + b \cosh(x)^2}}$$

Problem 18: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \cosh(x)^3} dx$$

Optimal(type 3, 182 leaves, 8 steps):

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a^{1/3} - b^{1/3}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{1/3} + b^{1/3}}}\right)}{3 a^{2/3} \sqrt{a^{1/3} - b^{1/3}} \sqrt{a^{1/3} + b^{1/3}}} + \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a^{1/3} + (-1)^{1/3} b^{1/3}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{1/3} - (-1)^{1/3} b^{1/3}}}\right)}{3 a^{2/3} \sqrt{a^{1/3} - (-1)^{1/3} b^{1/3}} \sqrt{a^{1/3} + (-1)^{1/3} b^{1/3}}}$$

$$+ \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a^{1/3} - (-1)^{2/3} b^{1/3}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{1/3} + (-1)^{2/3} b^{1/3}}}\right)}{3 a^{2/3} \sqrt{a^{1/3} - (-1)^{2/3} b^{1/3}} \sqrt{a^{1/3} + (-1)^{2/3} b^{1/3}}}$$

Result(type 7, 99 leaves):

$$\left(\sum_{R=\text{RootOf}((a-b)Z^6+(3a-3b)Z^4+(3a-3b)Z^2-a-b)} \frac{(-R^4+2R^2-1)\ln\left(\tanh\left(\frac{x}{2}\right)-R\right)}{3R^5a-R^5b-2R^3a-2R^3b+Ra-Rb} \right)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{1 - \cosh(x)^3} dx$$

Optimal(type 3, 71 leaves, 7 steps):

$$-\frac{2(-1)^{1/4} \arctan\left(\frac{(-1)^{3/4} \tanh\left(\frac{x}{2}\right) 3^{3/4}}{3}\right) 3^{1/4}}{3(1 - (-1)^{2/3})} - \frac{2(-1)^{1/4} \operatorname{arctanh}\left(\frac{(-1)^{3/4} \tanh\left(\frac{x}{2}\right) 3^{3/4}}{3}\right) 3^{1/4}}{3(1 + (-1)^{1/3})} - \frac{\sinh(x)}{3(1 - \cosh(x))}$$

Result(type 3, 211 leaves):

$$\begin{aligned} & \frac{3^{1/4} \sqrt{2} \arctan\left(\frac{\sqrt{2} 3^{3/4} \tanh\left(\frac{x}{2}\right)}{3} - 1\right)}{6} + \frac{3^{1/4} \sqrt{2} \ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2 + 3^{1/4} \tanh\left(\frac{x}{2}\right) \sqrt{2} + \sqrt{3}}{\tanh\left(\frac{x}{2}\right)^2 - 3^{1/4} \tanh\left(\frac{x}{2}\right) \sqrt{2} + \sqrt{3}}\right)}{12} + \frac{3^{1/4} \sqrt{2} \arctan\left(\frac{\sqrt{2} 3^{3/4} \tanh\left(\frac{x}{2}\right)}{3} + 1\right)}{6} \\ & - \frac{3^{3/4} \sqrt{2} \arctan\left(\frac{\sqrt{2} 3^{3/4} \tanh\left(\frac{x}{2}\right)}{3} - 1\right)}{18} - \frac{3^{3/4} \sqrt{2} \ln\left(\frac{\tanh\left(\frac{x}{2}\right)^2 - 3^{1/4} \tanh\left(\frac{x}{2}\right) \sqrt{2} + \sqrt{3}}{\tanh\left(\frac{x}{2}\right)^2 + 3^{1/4} \tanh\left(\frac{x}{2}\right) \sqrt{2} + \sqrt{3}}\right)}{36} \\ & - \frac{3^{3/4} \sqrt{2} \arctan\left(\frac{\sqrt{2} 3^{3/4} \tanh\left(\frac{x}{2}\right)}{3} + 1\right)}{18} + \frac{1}{3 \tanh\left(\frac{x}{2}\right)} \end{aligned}$$

Problem 20: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \cosh(x)^5} dx$$

Optimal(type 3, 312 leaves, 12 steps):

$$\begin{aligned}
& \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{1/5} - b^{1/5}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{1/5} + b^{1/5}}} \right)}{5 a^{4/5} \sqrt{a^{1/5} - b^{1/5}} \sqrt{a^{1/5} + b^{1/5}}} + \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{1/5} + (-1)^{1/5} b^{1/5}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{1/5} - (-1)^{1/5} b^{1/5}}} \right)}{5 a^{4/5} \sqrt{a^{1/5} - (-1)^{1/5} b^{1/5}} \sqrt{a^{1/5} + (-1)^{1/5} b^{1/5}}} \\
& + \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{1/5} - (-1)^{2/5} b^{1/5}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{1/5} + (-1)^{2/5} b^{1/5}}} \right)}{5 a^{4/5} \sqrt{a^{1/5} - (-1)^{2/5} b^{1/5}} \sqrt{a^{1/5} + (-1)^{2/5} b^{1/5}}} + \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{1/5} + (-1)^{3/5} b^{1/5}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{1/5} - (-1)^{3/5} b^{1/5}}} \right)}{5 a^{4/5} \sqrt{a^{1/5} - (-1)^{3/5} b^{1/5}} \sqrt{a^{1/5} + (-1)^{3/5} b^{1/5}}} \\
& + \frac{2 \operatorname{arctanh} \left(\frac{\sqrt{a^{1/5} - (-1)^{4/5} b^{1/5}} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{1/5} + (-1)^{4/5} b^{1/5}}} \right)}{5 a^{4/5} \sqrt{a^{1/5} - (-1)^{4/5} b^{1/5}} \sqrt{a^{1/5} + (-1)^{4/5} b^{1/5}}}
\end{aligned}$$

Result(type 7, 155 leaves):

$$\frac{1}{5} \left(\sum_{R=\operatorname{RootOf}(a-b)_Z^{10} + (-5a-5b)_Z^8 + (10a-10b)_Z^6 + (-10a-10b)_Z^4 + (5a-5b)_Z^2 - a - b} (-R^8 + 4R^6 - 6R^4 + 4R^2 - 1) \ln \left(\tanh\left(\frac{x}{2}\right) - R \right) \right) \frac{1}{R^9 a - R^9 b - 4R^7 a - 4R^7 b + 6R^5 a - 6R^5 b - 4R^3 a - 4R^3 b + Ra - Rb}$$

Problem 21: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \cosh(x)^6} dx$$

Optimal(type 3, 109 leaves, 7 steps):

$$\frac{\operatorname{arctanh} \left(\frac{a^{1/6} \tanh(x)}{\sqrt{a^{1/3} + b^{1/3}}} \right)}{3 a^{5/6} \sqrt{a^{1/3} + b^{1/3}}} + \frac{\operatorname{arctanh} \left(\frac{a^{1/6} \tanh(x)}{\sqrt{a^{1/3} - (-1)^{1/3} b^{1/3}}} \right)}{3 a^{5/6} \sqrt{a^{1/3} - (-1)^{1/3} b^{1/3}}} + \frac{\operatorname{arctanh} \left(\frac{a^{1/6} \tanh(x)}{\sqrt{a^{1/3} + (-1)^{2/3} b^{1/3}}} \right)}{3 a^{5/6} \sqrt{a^{1/3} + (-1)^{2/3} b^{1/3}}}$$

Result(type 7, 176 leaves):

$$\frac{1}{6} \left(\right)$$

$$\frac{\sum_{R=\text{RootOf}((a+b)z^{12}+(-6a+6b)z^{10}+(15a+15b)z^8+(-20a+20b)z^6+(15a+15b)z^4+(-6a+6b)z^2+a+b)} (-R^{10}+5R^8-10R^6+10R^4-5R^2+1) \ln\left(\tanh\left(\frac{x}{2}\right)-R\right)}{R^{11}a+R^{11}b-5R^9a+5R^9b+10R^7a+10R^7b-10R^5a+10R^5b+5R^3a+5R^3b-Ra+Rb}$$

Problem 22: Result is not expressed in closed-form.

$$\int \frac{1}{a+b \cosh(x)^8} dx$$

Optimal(type 3, 169 leaves, 9 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{(-a)^{1/8} \tanh(x)}{\sqrt{(-a)^{1/4}-b^{1/4}}}\right)}{4(-a)^{7/8} \sqrt{(-a)^{1/4}-b^{1/4}}} - \frac{\operatorname{arctanh}\left(\frac{(-a)^{1/8} \tanh(x)}{\sqrt{(-a)^{1/4}-1b^{1/4}}}\right)}{4(-a)^{7/8} \sqrt{(-a)^{1/4}-1b^{1/4}}} - \frac{\operatorname{arctanh}\left(\frac{(-a)^{1/8} \tanh(x)}{\sqrt{(-a)^{1/4}+1b^{1/4}}}\right)}{4(-a)^{7/8} \sqrt{(-a)^{1/4}+1b^{1/4}}} - \frac{\operatorname{arctanh}\left(\frac{(-a)^{1/8} \tanh(x)}{\sqrt{(-a)^{1/4}+b^{1/4}}}\right)}{4(-a)^{7/8} \sqrt{(-a)^{1/4}+b^{1/4}}}$$

Result(type 7, 232 leaves):

$$\frac{1}{8} \left(\sum_{R=\text{RootOf}((a+b)z^{16}+(-8a+8b)z^{14}+(28a+28b)z^{12}+(-56a+56b)z^{10}+(70a+70b)z^8+(-56a+56b)z^6+(28a+28b)z^4+(-8a+8b)z^2+a+b)} \left((-R^{14} + 7R^{12} - 21R^{10} + 35R^8 - 35R^6 + 21R^4 - 7R^2 + 1) \ln\left(\tanh\left(\frac{x}{2}\right)-R\right) \right) / \left((-R^{15}a + R^{15}b - 7R^{13}a + 7R^{13}b + 21R^{11}a + 21R^{11}b - 35R^9a + 35R^9b + 35R^7a + 35R^7b - 21R^5a + 21R^5b + 7R^3a + 7R^3b - Ra + Rb) \right) \right)$$

Problem 23: Result is not expressed in closed-form.

$$\int \frac{1}{1+\cosh(x)^5} dx$$

Optimal(type 3, 160 leaves, 11 steps):

$$\frac{\sinh(x)}{5(1+\cosh(x))} - \frac{2 \operatorname{arctan}\left(\frac{\tanh\left(\frac{x}{2}\right)}{\sqrt{\frac{-1+(-1)^{1/5}}{1+(-1)^{1/5}}}}\right)}{5\sqrt{-1+(-1)^{2/5}}} + \frac{2 \operatorname{arctanh}\left(\sqrt{\frac{1-(-1)^{4/5}}{1+(-1)^{4/5}}} \tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1+(-1)^{3/5}}} - \frac{2 \operatorname{arctan}\left(\sqrt{\frac{-1-(-1)^{3/5}}{1-(-1)^{3/5}}} \tanh\left(\frac{x}{2}\right)\right) \sqrt{\frac{-1-(-1)^{3/5}}{1-(-1)^{3/5}}}}{5(1+(-1)^{3/5})} + \frac{2 \operatorname{arctanh}\left(\sqrt{\frac{1-(-1)^{2/5}}{1+(-1)^{2/5}}} \tanh\left(\frac{x}{2}\right)\right)}{5\sqrt{1-(-1)^{4/5}}}$$

Result(type 7, 61 leaves):

$$\frac{\tanh\left(\frac{x}{2}\right)}{5} + \frac{\left(\sum_{R=\text{RootOf}(5Z^8+10Z^4+1)} \frac{(-5R^6+5R^4-5R^2+1)\ln\left(\tanh\left(\frac{x}{2}\right)-R\right)}{R^7+R^3} \right)}{50}$$

Problem 25: Result is not expressed in closed-form.

$$\int \frac{\tanh(x)^3}{a+b\cosh(x)^3} dx$$

Optimal(type 3, 112 leaves, 11 steps):

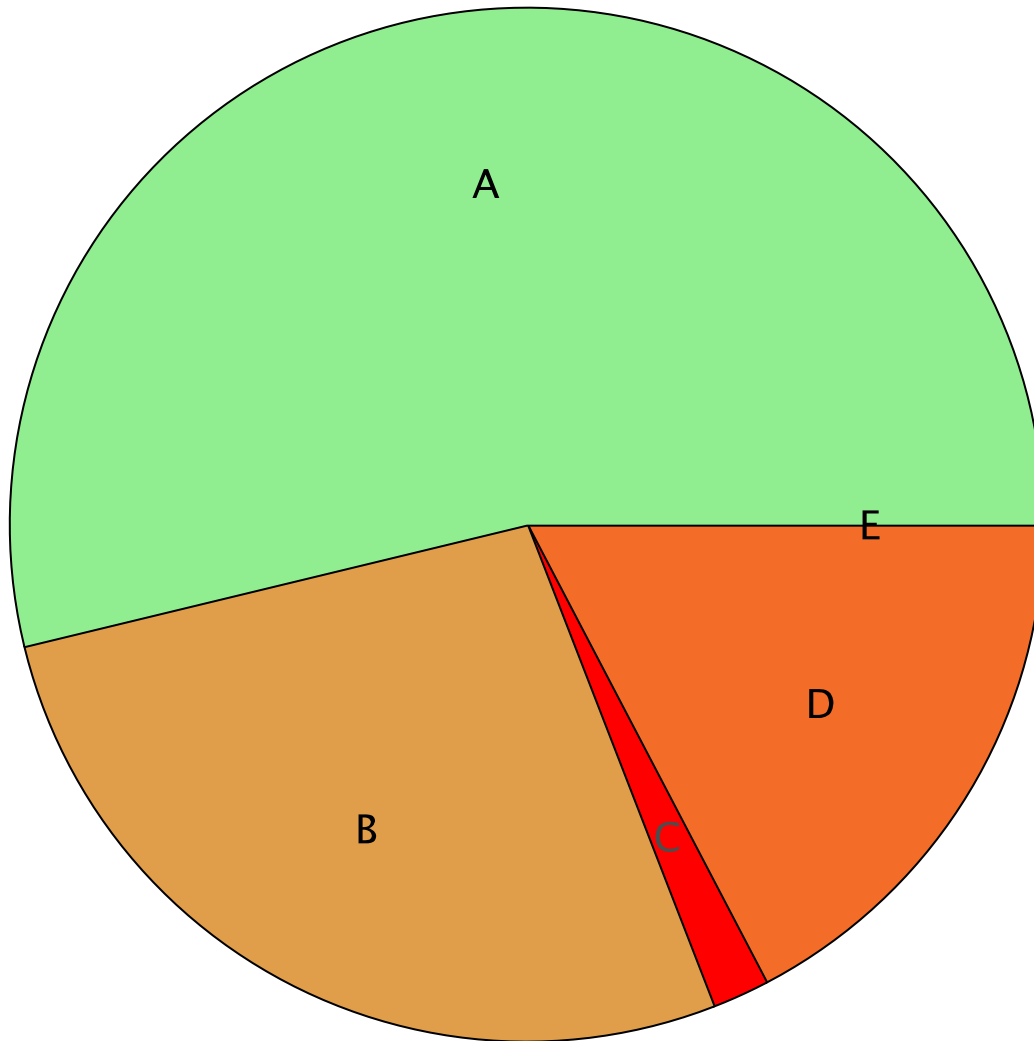
$$\frac{\ln(\cosh(x))}{a} + \frac{b^2/3 \ln(a^{1/3} + b^{1/3} \cosh(x))}{3a^{5/3}} - \frac{b^2/3 \ln(a^{2/3} - a^{1/3} b^{1/3} \cosh(x) + b^{2/3} \cosh(x)^2)}{6a^{5/3}} - \frac{\ln(a+b\cosh(x)^3)}{3a} + \frac{\text{sech}(x)^2}{2a} - \frac{b^2/3 \arctan\left(\frac{(a^{1/3} - 2b^{1/3} \cosh(x))\sqrt{3}}{3a^{1/3}}\right)\sqrt{3}}{3a^{5/3}}$$

Result(type 7, 149 leaves):

$$\sum_{R=\text{RootOf}((a-b)Z^3+(-3a-3b)Z^2+(3a-3b)Z-a-b)} \frac{(-R^2a - R^2b - 2Ra - 4Rb + a + b)\ln\left(\tanh\left(\frac{x}{2}\right)^2 - R\right)}{3a(-R^2a - R^2b - 2Ra - 2Rb + a - b)} + \frac{2}{a\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)}{a} - \frac{2}{a\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)}$$

Summary of Integration Test Results

225 integration problems



A - 121 optimal antiderivatives
B - 61 more than twice size of optimal antiderivatives
C - 4 unnecessarily complex antiderivatives
D - 39 unable to integrate problems
E - 0 integration timeouts